VOLUME 1

*Feynman*

# LECTURES ON PHYSICS

EXERCISES / 1964

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# FOREWORD

These exercises represent a first attempt at devising a suitable set of practice problems to accompany the Feynman Lectures on Physics. They were used in approximately their present form during the year 1962-63. Although the number of exercises is already considerably greater than even a superior student can expect to solve, there is still a distinct need for even more exercises, particularly of a kind that are numerically or analytically simple, yet incisive and illuminating in content. A few such exercises turn up each year, and as they do they will be incorporated in the set.

Exercises having a star with their number were taken from a compilation by Prof. Foster Strong, whose kind permission for their use is gratefully acknowledged.

R. B. Leighton
December 5, 1963

# CHAPTER 1

Use the ideas outlined in this Chapter, together with your own experience and imagination, in analyzing the following exercises: Precise numerical results are not expected.

1-1. Ordinary air has a density of about 0.001 g $cm^{-3}$, while liquid air has a density of about 1.0 g $cm^{-3}$.

   a) Estimate the number of air molecules per $cm^3$ in ordinary air and in liquid air.
   b) Estimate the mass of an air molecule.
   c) Estimate the average distance an air molecule should travel between collisions at normal temperatures and pressures (NTP). This distance is called the mean free path.
   d) Estimate at what pressure, in normal atmospheres, a vacuum system should be operated in order that the mean free path shall be about one meter.

1-2. A raindrop of an afternoon thundershower fell upon a Paleozoic mud flat and left an imprint which is later dug up as a fossil by a hot, thirsty geology student. As he drains his canteen, the student idly wonders how many molecules of water of that ancient raindrop he has just drunk. Estimate this number using only data which you already know. (Make reasonable assumptions regarding necessary information which you do not know.)

1-3. A glass full of water is left standing on an average outdoor windowsill in California.

   a) How long do you think it would take to evaporate completely?
   b) How many molecules $cm^{-2}s^{-1}$ would be leaving the water glass at this rate?
   c) Briefly discuss the connection, if any, between your answer to part a) and the average rainfall over the earth.

1-4. If the atoms of all objects are perpetually in motion, how can there be any permanent objects, such as fossil imprints?

1-5. Can you explain why there are no crystals which have the shape of a regular pentagon? (Triangles, squares, and hexagons are common in crystal forms).

1-6. How should the pressure P of a gas vary with n, the number of atoms per unit volume, and $\langle v \rangle$, the average speed of an atom? (Should P be proportional to n and/or $\langle v \rangle$, or should it vary more, or less, rapidly than linearly?)

1-7. If heat is merely molecular motion, what is the difference between a hot, stationary baseball and a cool, rapidly moving one?

1-8. Explain qualitatively why and how friction in a moving machine produces heat. Explain also, if you can, why heat cannot produce useful motion by the reverse process.

1-9. Chemists have found that the molecules of rubber consist of long criss-crossed chains of atoms. Explain why a rubber band becomes warm when it is stretched.

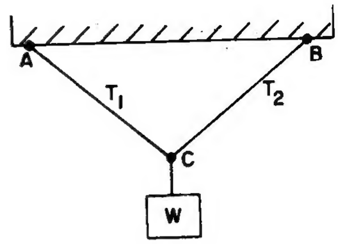
1-10. What should happen to a rubber band which is supporting a given weight, if it is heated? (To find out, try it.)

1-11. You are given a large number of steel balls of equal diameter d and a container of known volume v. Every dimension of the container is much greater than the diameter of a ball. What is the greatest number of balls that can be placed in the container?

# CHAPTER 4

Analyze the following exercises using the principle of conservation of energy or the principle of Virtual Work.

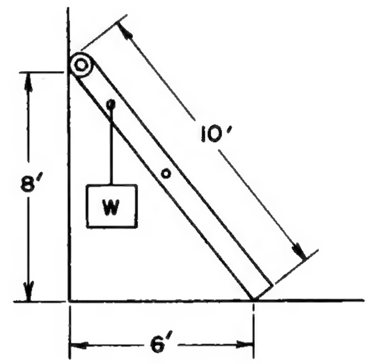
4-1. A weight $W = 50$ lb. is suspended from the midpoint of a wire ACB as shown. $AC = CB = 5$ ft. $AB = 5\sqrt{2}$ ft. Find the tension in the wire.



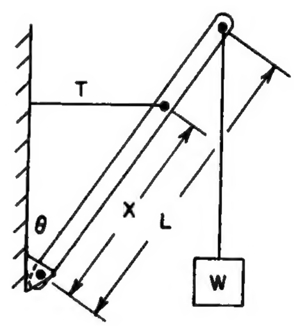

4-2. A uniform ladder 10 ft. long with rollers at the top end leans against a smooth vertical wall. The ladder weighs 30 lb. A weight $W = 60$ lb. is hung from a rung 2.5 feet from the top end. Find

a) The force with which the rollers push on the wall.

b) The horizontal and vertical forces with which the ladder pushes on the ground.




4-3. A derrick is made of a uniform boom of length L and weight w, pivoted at its lower end. It is supported at an angle $\Theta$ with the vertical by a horizontal cable attached at a point a distance x from the pivot, and a weight W is slung from its upper end. Find the tension in the horizontal cable.

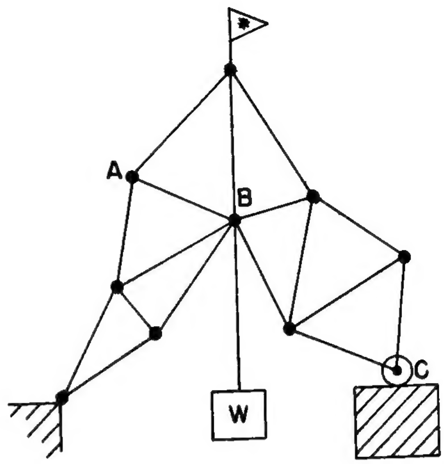



4-4. The truss in the figure is made of light aluminum struts freely pivoted

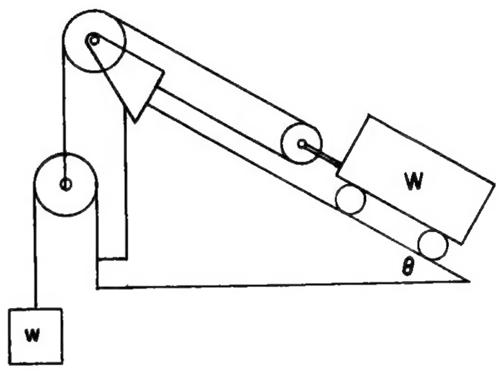
at each end. At C is a roller which rolls on a smooth plate. When a workman heats up member AB with a welding torch, it is observed to increase in length by an amount x, and the load W is thereby moved vertically an amount y.

a) Is the motion of W upward or downward?

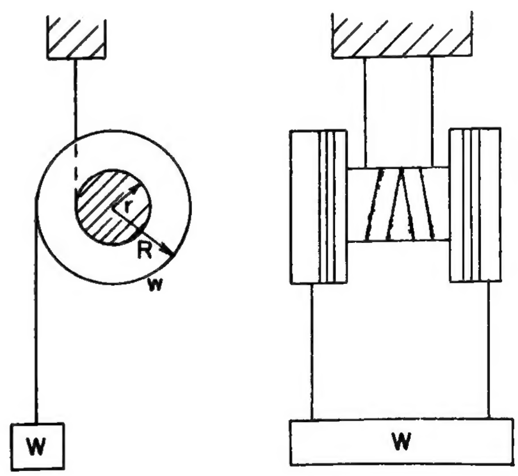
b) What is the force in the member AB (including the sense, i.e., tension or compression).



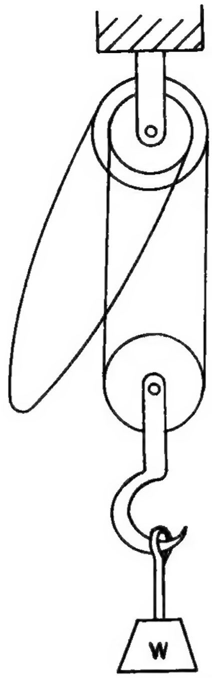

4-5. A cart on an inclined plane is balanced by the weight w. All parts have negligible friction. Find the weight W of the cart.




4-6. A spool of weight w has radii r and R. It is wound with cord, and suspended from a fixed support by two cords wound on the smaller radius. A weight W is then suspended from two cords wound on the larger radius, as shown. W is chosen so that the spool is just balanced. Find W.



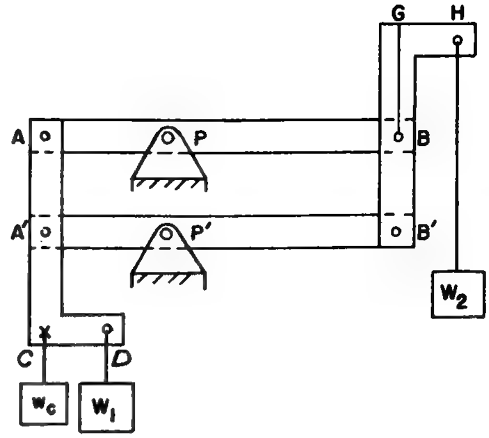
4-7.




The differential pulley shown uses chain with N links per ft. The upper pulley has indentations which match the chain links, the larger sheave having n notches in its circumference and the smaller sheave, n - 1 notches. (Each notch corresponds to <u>two</u> chain links.) The friction in the system is such that the ratio of the forces needed to raise or lower a weight W are in the ratio R. Assuming that the friction is the same in each direction find the forces needed to raise W and to lower W.

4-8. A loop of flexible chain, of total weight W, rests on a smooth, frictionless right circular cone of base radius r and height h. The chain rests in a horizontal circle on the cone, whose axis is vertical. Find the tension in the chain.

4-9. The jointed parallelogram frame AA'BB' is pivoted (in a vertical plane) on the pivots P and P'. There is negligible friction in the pins at A, A', B, B', P, and P'. The members AA'CD and B'BGH are rigid and identical in size.

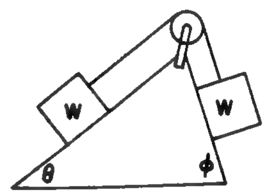



$$AP = A'P' = \tfrac{1}{2}PB = \tfrac{1}{2}P'B'$$

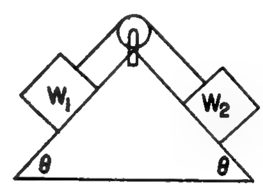
$$CD = GH = \tfrac{1}{2}AP$$

Because of the counterweight $w_c$, the frame is in balance without the loads $W_1$ and $W_2$. If a 0.50 kg weight $W_1$ is hung from D, what weight $W_2$, hung from H, is needed to produce equilibrium?

4-10. In the figure, the weights are equal, and there is no friction. If the system is released from rest, how fast are the weights moving when they have gone a distance D?

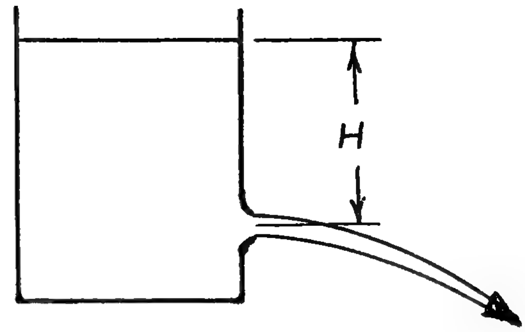



4-11. In the absence of friction, how fast will the weights $W_1$ and $W_2$ be going when they travel a distance D, starting from rest? $(W_1 > W_2)$

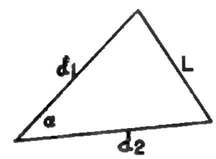



4-12. A tank of cross sectional area A contains a liquid having a density $\rho$.

The liquid squirts freely from a small hole of area $\alpha$ a distance H below the free surface of the liquid. If the liquid has no internal friction (viscosity), with what speed does it emerge?




4-13. In solving the problems up to this point, it should have become apparent that problems involving static equilibrium in the absence of friction may be reduced, using the Principle of Virtual Work, to problems of mere geometry: Where does one point move to when another moves a given small distance? In many cases this question is easily answered if the following properties of a triangle are used:




I. If the sides $d_1$ and $d_2$ remain fixed in length, but the angle $\alpha$ changes by a small amount $\Delta\alpha$, the opposite side L changes by an amount

$$\Delta L = \frac{d_1 d_2}{L} \sin \alpha \, \Delta\alpha$$

II. If the three sides a, b, c of a right triangle change in length by small amounts $\Delta a$, $\Delta b$, and $\Delta c$, then

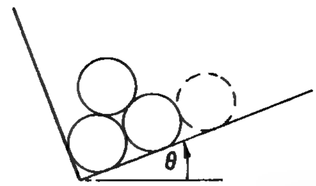
$$a\Delta a + b\Delta b = c\Delta c$$

(c is the hypotenuse)

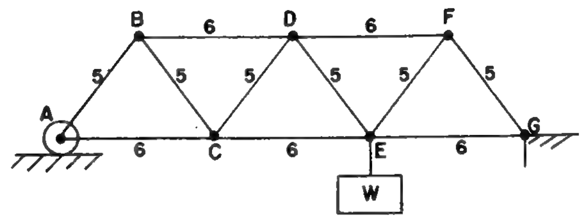
Can you prove these formulas?

4-14.* Smooth, identical logs are piled in a stake truck. The truck is forced off the highway and comes to rest on an even keel lengthwise

but with the bed at an angle Θ with the horizontal. As the truck is unloaded, the removal of the log shown dotted leaves the remaining three in a condition where they are just ready to slide, that is, if Θ were any smaller, the logs would fall down. Find Θ.



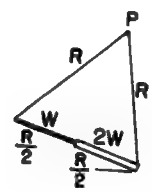

4-15. In the truss shown, all diagonal struts are of length 5 units and all horizontal ones are of length 6 units. All joints are freely hinged, and the weight of the truss is negligible.



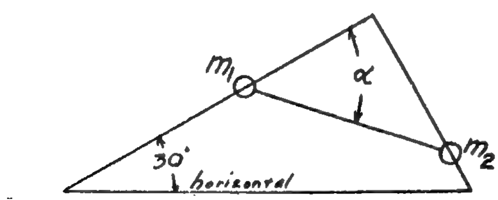

a) Which of the members could be replaced with flexible cables, for the load position shown?

b) Find the forces in struts BD, and DE.

4-16.* A rod of length R is made of two uniform pieces of equal length R/2 each, but one piece weighs twice as much as the other. The rod is suspended by cords of length R attached to each end and to a nail at P. When the system comes to rest, what angle α does the rod make with the horizontal?




4-17.*




A rigid wire frame is formed in a right triangle, and set in a vertical plane as shown. Two beads of masses $m_1 = 100$ gram, $m_2 = 300$ gram slide without friction on the wires, and are connected by a cord. When the system is in static equilibrium, what is the tension in the cord, and what angle α does it make with the first wire?

# CHAPTER 6

6-1. An air molecule at 25° C and 760 mm pressure travels about $7 \times 10^{-6}$ cm between successive collisions and moves with a mean speed of about 450 m $sec^{-1}$. In the absence of any <u>bodily</u> motion of the air, about how long should it take for a given molecule to move 1 cm from where it is now?

6-2. A boy has a bag which contains two red marbles, three green marbles, and one white marble. He takes three marbles at random out of the bag. What is the probability that they are

a) All of different colors, and

b) All of the same color?

6-3. Small B-B's of radius <u>a</u> are shot at random at a fixed sphere of radius <u>b</u>. Assume that the collisions are perfectly elastic and that the angle of incidence is equal to the angle of rebound (measured from the line of centers of the spheres when they are in contact). Derive an expression for the relative numbers of B-B's that will be deflected through various angles, and express the result as a cross-section. Be sure your expression reduces to the obvious value $\pi(a + b)^2$ for zero deflection.

## APPENDIX

Some properties of the ellipse:

The size and shape of an ellipse are determined by specifying the values of any two of the following quantities:

- $a$ : the semi major axis
- $b$ : the semi minor axis
- $c$ : the distance from the center to one focus
- $e$ : the eccentricity
- $r_p$ : the perihelion distance (the closest distance from a focus to the ellipse)
- $r_a$ : the aphelion distance (the farthest distance from a focus to the ellipse)

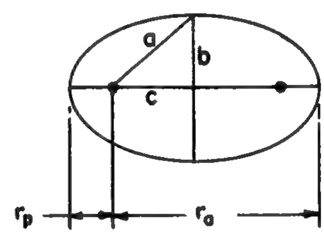
The relationships of these various quantities are as follows:

$$a^2 = b^2 + c^2$$

$$e = c/a \text{ (Definition of e)}$$

$$r_p = a - c = a(1 - e)$$

$$r_a = a + c = a(1 + e)$$

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7-1. The distance of the moon from the center of the Earth varies from 363,300 km at perigee to 405,500 km at apogee, and its period is $27^d.322$. A certain artificial earth satellite is orbiting so that its perigee height from the surface of the Earth is 225 km, and its apogee height is 710 km. The mean diameter of the Earth is 12,756 km. What is the sidereal period of this satellite?

7-2. A satellite of mass m moves in a circular orbit of radius R about a much larger attracting object of mass M.

a) Making use of the relation $s = \frac{1}{2} at^2$ and the argument given in the Chapter, derive an expression for the acceleration, toward the center, which the satellite experiences while moving in its (circular) path. Express this acceleration in terms of orbital speed and orbital radius.

b) Assuming $ma = G \frac{Mm}{R^2}$, derive Kepler's third law.

7-3. a) Comparing data describing the Earth's orbital motion about the sun with data for the moon's orbital motion about the Earth, determine the mass of the sun relative to the mass of the Earth. (Use relations derived in the previous problem.)

b) Io, a moon of Jupiter, has an orbital period of revolution of $1^{d}.769$ and an orbital radius of 421,800 km. Determine the mass of Jupiter in terms of the mass of the Earth.

7-4. Using the idea that two mutually gravitating bodies each "fall" toward the other, and thus move about some fixed common point (their center of mass), show that their period in an orbit in which they remain a given, fixed distance R apart, depends only upon the sum of their masses and not at all upon the ratio of their masses. This is also true for elliptical orbits. Try to prove it.

7-5. Two stars, a and b, move around one another under the influence of their mutual gravitational attraction. If the semi major axis of their relative orbit is observed to be R, measured in astronomical units (A.U.) and their period of revolution is T years, find an expression for the sum of the mass, $m_a + m_b$, in terms of the mass of the sun.

7-6. The trigonometric parallax of Sirius (i.e., the angle subtended at Sirius by the radius of the Earth's orbit) is 0".378 arc. Using this and the data contained in Fig. 7-7, deduce as best you can the mass of the Sirius system in terms of that of the sun, a) assuming that the orbital plane is perpendicular to the line of sight, and b) allowing for the actual tilt of the orbit. Is your value in part b) an upper or lower limit (or either)?

7-7. The eccentricity of the Earth's orbit is 0.0167. Find the ratio of its maximum speed in its orbit to its minimum speed.

7-8. In 1986, Halley's comet is expected to return on its seventh trip around the sun since the days in 1456 when people were so frightened that they offered prayers in the churches "to be saved from the Devil, the Turk, and the comet." In its most recent perihelion on April 19, 1910, it was observed to pass near the sun at a distance of 0.60 A.U.

a) How far does it go from the sun at the outer extreme of its orbit?

b) What is the ratio of its maximum orbital speed to its minimum speed?

7-9. How can one find the mass of the moon?

7-10. The radii of the Earth and the moon are 6378 km and 1738 km, respectively, and their masses are in the ratio 81.3 to 1.000. Calculate the acceleration of gravity at the surface of the moon. $g_{\oplus} = 9.80 \text{ m sec}^{-2}$.

7-11. In making laboratory measurements of g, how precise does one have to be to detect diurnal variations in g due to the moon's gravitation? For simplicity, assume that your laboratory is so located that the moon passes through zenith and nadir. Also, neglect earth-tide effects.

# CHAPTER 8

8-1. A body travels in a straight line with a constant acceleration a. At $t = 0$, it is located at $x = x_o$ and has a velocity $v_x = v_{xo}$. Show that its position and velocity at time t are

$$x(t) = x_o + v_{xo}t + \tfrac{1}{2} at^2$$

$$v_x(t) = v_{xo} + at$$

8-2. Eliminate t from the preceding equations, and thus show that, at any time,

$$v_x^2 = v_{xo}^2 + 2a(x - x_o)$$

8-3. Generalize the preceding problems to the case of three-dimensional motion with constant acceleration components, $a_x$, $a_y$, $a_z$, along the three coordinate axes.

8-4. A projectile is fired over level terrain at initial speed V, at an angle $\theta$ with the horizontal. (Neglect air resistance.) Find the maximum height attained and the range.

8-5. At what angle should the above projectile be fired in order to attain the maximum range?

8-6.* A Caltech freshman, inexperienced with suburban traffic officers, has just received a ticket for speeding. Thereafter, when he comes upon one of the "Speedometer Test" sections on a level stretch of highway, he decides to check his speedometer reading. As he passes the "0" start of the marked section, he presses on his accelerator and for the entire period of the test he holds his car at constant acceleration. He notices that he passes the 0.10 mile post 16 sec.

after starting the test, and 8.0 sec. later he passes the 0.20 mile post.

a) What should his speedometer have read at the 0.2 mile post?

b) What was his acceleration?

8-7.* A Corporal rocket fired vertically is observed to have a constant upward acceleration of 2 g during the burning of the rocket motor, which lasted for 50 seconds. Neglecting air resistance and variation of g with altitude,

a) Draw a v-t diagram for entire flight of rocket;

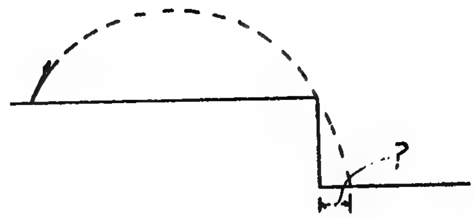
b) calculate the maximum height attained;

c) calculate the total elapsed time from the firing of the rocket to its return to Earth.

8-8.* On the long horizontal test track at Edwards AFB, both rocket and jet motors can be tested. On a certain day, a rocket motor, started from rest, accelerated constantly until its fuel was exhausted, after which it ran at constant speed. It was observed that this exhaustion of the rocket fuel took place as the rocket passed the mid-point of the measured test distance. Then a jet motor was started from rest down the track, with a constant acceleration for the entire distance. It was observed that both rocket and jet motors covered the test distance in exactly the same time. What was the ratio of the acceleration of the jet motor to that of the rocket motor?

8-9.* A mortar emplacement is set 27,000 ft horizontally from the edge of a bluff that drops 350 ft down from the level of the mortar.

It is desired to shell objects concealed on the ground behind the bluff. How close to the bottom edge of the cliff can shells reach if fired at a muzzle speed of 1000 ft/sec?




8-10. An angle may be measured by the length of arc of a circle that the angle subtends, with the vertex of the angle at the center of the circle. If s is the arc length and R is the radius of the circle, then the subtended angle $\theta$, in radians, is

$$\theta = s/R$$

a) Show that, if $\theta \ll 1$ radian, $\sin\theta \approx \theta$, and $\cos\theta \approx 1$.

b) With the above result, and the formulas for the sine and cosine of the sum of two angles, find the derivatives of sin x and cos x, using the fundamental formula

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{y(x + \Delta x) - y(x)}{\Delta x}$$

8-11. An object is moving counterclockwise in a circle of radius R at constant speed V. The center of the circle is at the origin of rectangular coordinates (x,y), and at t = 0 the particle is at (R,0).

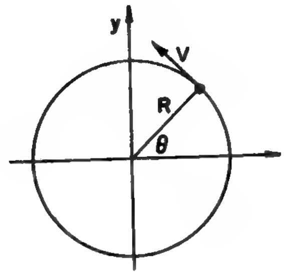
a) Find $x$, $y$, $v_x$, $v_y$, $a_x$, and $a_y$ as functions of time.

b) Show that

$$\ddot{x} + \omega^2 x = 0$$

and $\ddot{y} + \omega^2 y = 0$

where $\omega = V/R$




8-12. A small pebble is lodged in the tread of a tire of radius R. If this tire is rolling at speed V without slipping on a horizontal road, find the equations for the x and y coordinates of the pebble as a function of time. Let the pebble touch the road at t = 0. Find also the velocity and acceleration components as a function of the time.

# CHAPTER 9

9-1. A particle of mass m is moving in a region where it experiences a force proportional to its velocity and directed at right angles both to its velocity and to a certain fixed line, say the z-axis. If the particle is initially moving in the x-y plane at speed V, show that it moves in a circular path and find the radius of the path. Let the proportionality constant for the force be $\beta$, so $F = \beta v$.

9-2. Find the radii of curvature of the orbit of Fig. 9-6 at $t = 0$, $t = .82$, and $t = 2.086$ sec.

9-3. A boy throws a ball upward at an angle of $70^\circ$ with the horizontal, and it passes neatly through an open window, 32 ft above his shoulder, moving horizontally.

a) How fast was the ball moving as it left his hand?

b) What was the radius of curvature of its path as it passed over the windowsill?

Can you find the radius of curvature of its path at any given time?

9-4. Moe and Joe, two cosmic physicists who grew up on different planets, meet at an interplanetary symposium on weights and measures to discuss the establishment of a universal system of units. Moe proudly describes the merits of the MKSA system, used in every civilized region of the Earth. Joe equally proudly describes the beauties of the M'K'S'A' system, used everywhere else in the solar system. If the constant factors relating the basic mass, length, and time standards of the two systems are $\mu$, $\lambda$, and $\tau$, such that

$$m' = \mu m, \quad l' = \lambda l, \quad \text{and } t' = \tau t$$

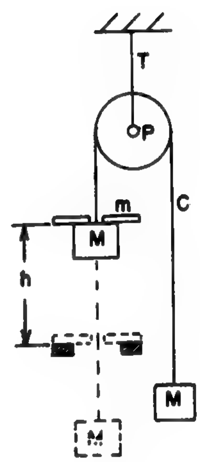
what factors are needed to convert the units of velocity, acceleration, force, and energy between the two systems?

9-5. How will the numerical magnitudes of the constant of gravitation measured in the two systems of units described in the preceding problem, be related?

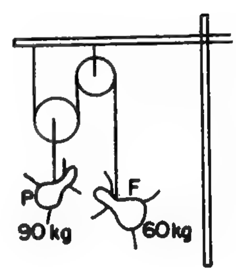
9-6. What is the numerical magnitude of $GM_{\odot}$ if lengths are measured in A.U. and times in years?

9-7. If a scale model of the solar system is made, using materials of the same respective average densities as the sun and planets, but reducing all linear dimensions by a scaling factor k, how will the periods of revolution of the planets depend upon k?

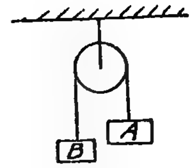
9-8. An early arrangement for measuring the acceleration of gravity, called Atwood's Machine, is shown in the figure. The pulley P and cord C have negligible mass and friction. The system is balanced with equal masses M on each side, and then a small rider m is added to one side. After accelerating through a certain distance h, the rider is caught on a ring and the two equal masses then move on with constant speed v. Find the value of g that corresponds to the measured values of m, M, h, and v.




9-9. None of the identical gondolas on the Martian canal Rimini is quite able to support the load of both Paolo and Francesca, two affectionate marsupials who refuse to go in separate boats. The enterprising gondolier, Guiseppi, collects their fare by rigging them up from the mast as in the figure, using the massless ropes and massless, frictionless pulleys characteristic of Martian construction. Giuseppi ferries them across before they hit either the mast or the deck. How much load does he save? Hint: Remember that the tension in a massless cord that passes over a massless, frictionless pulley is the same on both sides of the pulley.



9-10.* A space traveler about to leave for the moon has a spring balance and a 1.0 lb mass A, which when hung on the balance on the Earth gives the reading of 1.0 lb of force. Arriving at the moon at a place where the acceleration of gravity is not known exactly but has a value of about 1/6 the acceleration of gravity at the Earth's surface, he picks up a stone B which gives a reading of 1.0 lb of force when weighed on the spring balance. He then hangs A and B over a pulley as shown in the figure and observes that B falls with an acceleration of 4.0 ft/sec$^2$. What is the mass of stone B?



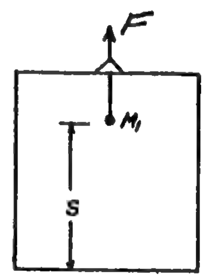

9-11.* An elevator of mass $M_2$ has hanging from its ceiling a mass $M_1$. The elevator is being accelerated upward by a constant force F (F greater than $(M_1 + M_2)g$). The mass $M_1$ is initially a distance $\underline{s}$ above the elevator floor.

a) Find the acceleration of the elevator.

b) What is the tension in the string connecting the mass $M_1$ to the elevator?

c) If the string suddenly breaks, what is the acceleration of the elevator immediately after? What is the acceleration of mass $M_1$?

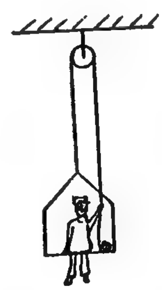
d) How long does it take for $M_1$ to hit the bottom of the elevator?



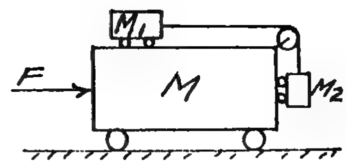

9-12.* A painter weighing 180 lb working from a "bosun's" chair hung down the side of a tall building, desires to move in a hurry. He pulls down on the fall rope with such a force that he presses against the chair with only a force of 100 lb. The chair itself weighs 30.0 lb.

a) What is the acceleration of the painter and the chair?

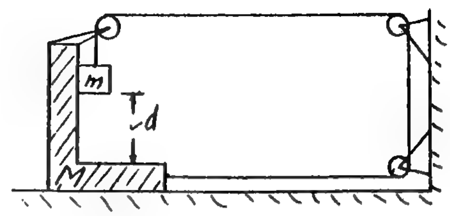
b) What is the total force supported by the pulley?



9-13.* What horizontal force F must be constantly applied to M so that $M_1$ and $M_2$ do not move relative to M?




9-14.* Given the system shown--all surfaces are frictionless. If m = 150 g is released when it is d = 4.0 ft above the base of M = 1650 g, how long after release will m strike the base of M?




9-15. A mass suspended from a spring hangs motionless, and is then given an upward blow such that it moves initially at unit speed. If the mass and spring constant are such that the equation of motion is $\ddot{x} = -x$, find the maximum height attained by numerical integration of the equation of motion.

9-16. A particle of mass m moves along a straight line. Its motion is resisted by a force proportional to its velocity, $F = -kv$. If it starts with speed $v = v_o$ at $x = 0$ and $t = 0$, find x as a function of t by numerical integration. Find the time $t_{\frac{1}{2}}$ required to lose half its speed, and the maximum distance $x_m$ attained.

NOTE:
a) Adjust the scales of x and t so that the equation of motion has simple numerical coefficients.
b) Invent a scheme analogous to that introduced in the text to attain good accuracy with a relatively coarse interval for $\Delta t$.
c) Use dimensional analysis to deduce how $t_{\frac{1}{2}}$ and $x_m$ should depend upon $v_o$, k, and m, and solve for the actual motion only for a single convenient value of $v_o$, say $v_o = 1.00$ (in the modified x and t units).

9-17. A certain charged particle moves in an electric and a magnetic field

according to the equations

$$\frac{dv_x}{dt} = - 2v_y$$

$$\frac{dv_y}{dt} = 1 + 2v_x$$

At $t = 0$, the particle starts at $(0,0)$ with velocity $v_x = 1.00$, $v_y = 0$. Determine the nature of the motion by numerical integration. See NOTE (b) in the preceding exercise.

9-18. A shell is fired with a muzzle velocity of 1000 ft sec$^{-1}$ at an angle of $45^\circ$ with the horizontal. Its motion is resisted by a force proportional to the cube of its velocity ($F = - kv^3$). The coefficient k is such that the resisting force is equal to twice the weight of the shell when $v = 1000$ ft sec$^{-1}$. Find the approximate maximum height attained and the horizontal range by numerical integration, and compare these with the values expected in the absence of resistance.

# CHAPTER 10

10-1. Two gliders are free to move in a horizontal air trough. One is stationary and the other collides with it perfectly elastically. They rebound with equal and opposite velocities. What is the ratio of their masses?

10-2. Two equally massive gliders, moving in a level air trough at equal and opposite velocities, v and -v, collide almost elastically, and rebound with slightly smaller speeds. They lose a fraction $f \ll 1$ of their kinetic energy in the collision. If these same gliders collide with one of them initially at rest, with what speed will the second glider move after the collision? (This small residual speed $\Delta v$ may easily be measured in terms of the final speed v of the originally stationary glider, and thus the elasticity of the spring bumpers may be determined.)

NOTE: If $x \ll 1$, $\sqrt{1 - x} \approx 1 - \frac{1}{2}x$.

10-3. An earth satellite of mass 10 kg and average cross-sectional area 0.50 $m^2$ is moving in a circular orbit at 200 km altitude where the molecular mean free paths are many meters and the air density is about $1.6 \times 10^{-10}$ kg $m^{-3}$. Under the crude assumption that the molecular impacts with the satellite are effectively inelastic (but that the molecules do not literally stick to the satellite but drop away from it at low relative velocity), calculate the retarding force that the satellite would experience due to air friction. How should such a frictional force vary with velocity? Would the satellite's speed decrease as a result of the net force on it? (Check the speed of a circular satellite orbit vs. height.)

10-4. A rocket of initial mass $M_o$ kg ejects its burnt fuel at a constant rate $dm/dt = - r_o$ kg sec$^{-1}$ and at a velocity $V_o$ (relative to the rocket)

a) Calculate the initial acceleration of the rocket (neglect gravity)

b) If $V_o = 2.0$ km sec$^{-1}$, how many kilograms of fuel must be ejected per second to develop $10^5$ kgwt of thrust?

c) Write a differential equation which connects the speed of the rocket with its residual mass, and solve the equation if you can.

10-5. When two bodies move along a line, there is a special system of coordinates in which the momentum of one body is equal and opposite to that of the other. That is, the total momentum of the two bodies is zero. This frame of reference is called the center-of-mass system (abbreviated CM). If the bodies have masses $m_1$ and $m_2$ and are moving at velocities $v_1$ and $v_2$, show that the CM system is moving at velocity

$$v_{CM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

10-6. Generalize Ex. 10-5 to any number of masses moving along a line, i.e., find the velocity of the coordinate system in which the total momentum is zero.

10-7. If T is the total kinetic energy of the two masses in Ex. 10-5, and $T_{CM}$ is their total kinetic energy in the CM system, show that

$$T = T_{CM} + \tfrac{1}{2}(m_1 + m_2)V_{CM}^{\,2}$$

10-8. Can you generalize the result of Ex. 10-7 to any number of masses?

10-9. A neutron having a kinetic energy E collides head-on with a stationary nucleus of $C^{12}$ and rebounds perfectly elastically in the direction from which it came. What is its final kinetic energy?

10-10. The speed of a rifle bullet may be measured by means of a ballistic pendulum: The bullet, of known mass m and unknown speed V, embeds itself in a stationary wooden block of mass M, suspended as a pendulum of length L. This sets the block to swinging. The amplitude x of swing may be measured and, using conservation of energy, the velocity of the block immediately after impact may be found. Derive an expression for the speed of the bullet in terms of m, M, L, and x.

# CHAPTER 11

11-1. If vectors $\vec{a} = 3\vec{i} + 2\vec{j} - \vec{k}$, $\vec{b} = 2\vec{i} - \vec{j} + \vec{k}$, and $\vec{c} = \vec{i} + 3\vec{j}$, find

a) $\vec{a} + \vec{b}$

b) $\vec{a} - \vec{b}$

c) $a_x$

Ans: $5\vec{i} + \vec{j}$

d) $\vec{a} \cdot \vec{i}$

e) $\vec{a} \cdot \vec{b}$

f) $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

11-2.* A cyclist rides at 10 mi hr$^{-1}$ due north and the wind (which is blowing at 6 mi hr$^{-1}$ from a point between N and E) appears to the cyclist to come from a point 15$^{\circ}$ E of N.

a) Find the true direction of the wind;

b) the direction in which the wind will appear to meet him on his return if he rides at the same speed.

11-3.* You are on a ship traveling steadily east at 15 knots. A ship on a steady course whose speed is known to be 26 knots is observed 6.0 mi due south of you, it is later observed to pass behind you, its distance of closest approach being 3.0 mi.

a) What was the course of the other ship?

b) What was the time between its position south of you and its position of closest approach?

11-4.* A rigid wheel of radius R is rolling without slipping on a horizontal surface. The plane of the wheel is vertical, and the axis of the wheel is moving horizontally with a speed V relative to the surface. Calculate the instantaneous velocity (speed and direction) of any point on the rim of the wheel.

11-5.* A motorboat that runs at a constant speed V relative to the water is operated in a straight river channel where the water is flowing smoothly with a constant speed R. The boat is first sent on a round

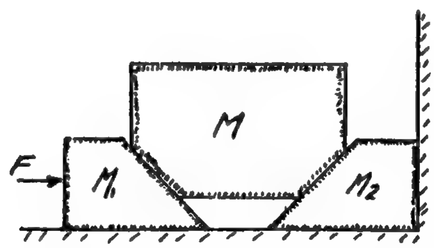
trip from its anchor point to a point a distance d directly upstream. It is then sent on a round trip from its anchor point to a point a distance d away directly across the stream. For simplicity, assume that the boat runs the entire distance in each case at full speed and that no time is lost in reversing course at the end of the outward lap. If $t_V$ is the time the boat took to make the round trip in line with the stream flow, $t_A$ the time the boat took to make the round trip across the stream, and $t_L$ the time the boat would take to go a distance 2d on a lake,

a) What is the ratio $t_V/t_A$?

b) What is the ratio $t_A/t_L$?

11-6.* A man standing on the bank of a river 1.0 mi wide wishes to get to a point directly opposite him on the other bank. He can do this in two ways: (1) head somewhat upstream so that his resultant motion is straight across, (2) head toward the opposite bank and then walk up along the bank from the point downstream to which the current has carried him. If he can swim 2.5 mi $hr^{-1}$ and walk 4.0 mi $hr^{-1}$, and if the current is 2.0 mi $hr^{-1}$, which is the faster way to cross, and by how much?

11-7.*




Two identical 45° wedges $m_1$ and $m_2$, with smooth faces and $m_1 = m_2 = 8.0$ kg, are used to move a smooth-faced mass M = 384 kg. Both wedges rest upon a smooth horizontal plane; one wedge is butted against a vertical wall, and to the other wedge a force F = 592 kgwt is applied horizontally.

a) What is the magnitude and direction of the acceleration of the movable wedge $m_1$?

b) What is the magnitude and direction of the acceleration of the larger wedge M?

c) What force does the stationary wedge $m_2$ exert on the heavy mass M?

11-8. A mass m is suspended from a frictionless pivot at the end of a string of arbitrary length, and is set to whirling in a horizontal circular path whose plane is a distance H below the pivot point. Find the period of revolution of the mass in its orbit.

11-9. Generalize Ex. 10-5 to 10-8 to three dimensional motion using vector notation.

Let $M = \sum_{i=1}^{N} m_i$

11-10. A "particle" of mass $m_1 = 2$ kg, moving with a velocity $\vec{v}_1 = 3\vec{i} + 2\vec{j} - \vec{k}$ m sec$^{-1}$ collides inelastically with a second particle of mass $m_2 = 3$ kg, moving with a velocity $\vec{v}_2 = -2\vec{i} + 2\vec{j} + 4\vec{k}$ m sec$^{-1}$. Find the velocity of the composite particle.

11-11. Find the kinetic energy in the CM system of the above particles before impact.

11-12. A puck of mass 1 kg, moving at a speed of 6 m sec$^{-1}$ due N collides with a stationary puck of mass 2 kg. After the collision, the 1 kg puck is moving at 45° NE of its original direction, at a speed of $2\sqrt{2}$ m sec$^{-1}$.

a) What is the velocity of the 2 kg puck after impact?

b) What fraction of the kinetic energy was lost in the CM system?

c) Through what angle was the 1 kg puck deflected in the CM system?

The following approach to the analysis of two-body collisions is often a useful one:

1) Find the CM velocity $\vec{V}_{CM}$

2) Subtract $\vec{V}_{CM}$ from $\vec{V}_1$ and $\vec{V}_2$ (the velocities of bodies 1 and 2 before collision) to obtain the CM velocities $\vec{V}_1{}'$ and $\vec{V}_2{}'$.

3) The momenta of 1 and 2 are now equal and opposite.

4) The collision now takes place, and its effect is:

   a) to rotate the line of relative motion of 1 and 2

   b) to increase, decrease, or leave unchanged, the magnitudes of $\vec{V}_1{}'$ and $\vec{V}_2{}'$, according to whether

energy is released, absorbed, or unchanged in the collision.

5) Add $V_{CM}$ back on to the final CM velocities $\vec{U}_1{}'$ and $\vec{U}_2{}'$ to obtain the final velocities $\vec{U}_1$ and $\vec{U}_2$.

11-13. A moving particle collides perfectly elastically with an equally massive particle initially at rest. Show that the two particles move at right angles to one another after the collision.

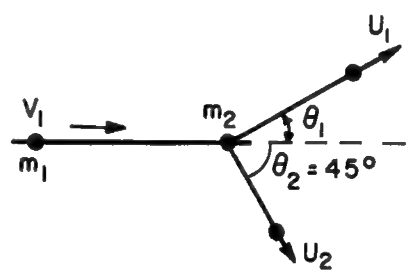
11-14. Two small putty balls, A and B, each of mass 1 gram, travel under the influence of gravity with acceleration $-9.8\,\vec{k}$ m/sec$^2$. Given the initial conditions:

at $t = 0$, $\vec{r}_a(0) = 7\vec{i} + 4.9\vec{k}$ (meters); $\vec{V}_a(0) = 7\vec{i} + 3\vec{j}$ (m sec$^{-1}$)

$\vec{r}_b(0) = 49\vec{i} + 4.9\vec{k}$ (meters); $\vec{V}_b(0) = -7\vec{i} + 3\vec{j}$ (m sec$^{-1}$)

Find $\vec{r}_a(t)$ and $\vec{r}_b(t)$ for all times $t > 0$ .

11-15. A particle of mass $m_1$ and velocity $\vec{v}_1$ collides perfectly elastically with another particle of mass $m_2 = 3m_1$ which is at rest ($\vec{v}_2 = 0$). After the collision, $m_2$ moves at angle $\theta_2 = 45^\circ$ with respect to the original direction of $m_1$.




a) Find $\theta_1$, the angle of continuation of $m_1$, and $\vec{U}_1$, $\vec{U}_2$ the final velocities.

11-16. A moving particle of mass M collides perfectly elastically with a stationary particle of mass $m < M$. Find the maximum possible angle through which the incident particle can be deflected.

11-17. A particle of mass m collides perfectly elastically with a stationary particle of mass $M > m$. The incident particle is deflected through a $90^\circ$ angle. At what angle $\theta$ with the original direction of m does the more massive particle recoil?

11-18. In the collision of the preceding problem, a fraction $1 - \alpha^2$ of the CM energy is lost. What now is the recoil angle $\Theta$ of the originally stationary particle?

11-19. A particle of mass 1 kg is moving in such a way that its position is described by the vector

$$\vec{r} = t\,\vec{i} + (t + t^2/2)\vec{j} - (4/\pi^2)\sin \pi t/2\,\vec{k}$$

a) Find the position, velocity, acceleration, and kinetic energy of the particle at $t = 0$ and $t = 1$ sec.

b) Find the force which will produce this motion.

c) Find the radius of curvature of the particle's path at $t = 1$ sec.

11-20. A particle is initially at a point $\vec{r}_0$, and is moving under gravity with an initial velocity $\vec{v}_0$. Find the subsequent motion.

11-21. Use vectors to find the great circle distance between two points on the earth whose latitudes and longitudes are $(\lambda_1, \Phi_1)$ and $(\lambda_2, \Phi_2)$.

NOTE: Use a system of rectangular coordinates with origin at the center of the earth, one axis along the earth's axis, another pointed toward $\lambda = 0$, $\Phi = 0$, and the third axis pointed toward $\lambda = 0$, $\Phi = 90^\circ$ W. (Let longitudes vary from $0^\circ$ westward to $360^\circ$.)

11-22. What is the magnitude and direction of the acceleration of the moon at

a) new moon,

b) quarter moon,

c) full moon?

NOTE: $R_{\oplus\odot} = 1.50 \times 10^8$ km,

$R_{\oplus ☽} = 3.85 \times 10^5$ km,

$M_\odot = 3.33 \times 10^5\, M_\oplus$

12-1. A block of mass m slides on an inclined plane tilted at an angle $\theta$ with the horizontal. If the coefficient of sliding friction is $\mu < \mathrm{Tan}\,\theta$, with what acceleration does the block move

a) up the plane?

b) down the plane?

c) at an angle $\Phi$ measured in the plane from a horizontal line in the plane?

(Use x and y coordinates in the plane, with x horizontal and y up the plane)

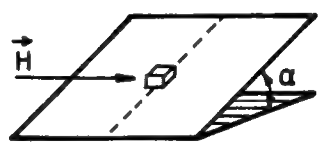
12-2. In the previous problem, let $m = 1.00$ kg, $\mu = 0.20$ and $\theta = 30^{\circ}$. If the block is projected up the plane at an initial speed $3.00\ \mathrm{m\ sec}^{-1}$

a) How far up the plane does it go?

b) How long does it take to go up and back to its starting point?

c) How much energy is lost to heat in the process?

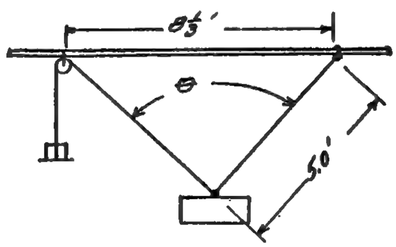
12-3. A particle of weight W rests on a rough inclined plane which makes an angle $\alpha$ with the horizontal.




a) If the coefficient of static friction $\mu = 2\tan\alpha$, find the least horizontal force $H_{min}$, acting transverse to the slope of the plane which will cause the particle to move.

b) In what direction will it go?

12-4.*




A mass of 1000 grams is supported by a cord 5.0 feet long fastened to a ring free to move on a horizontal rod. The coefficient of static friction between the ring and the rod is 0.75. A second cord is fastened to the weight and passes over a pulley fastened to the

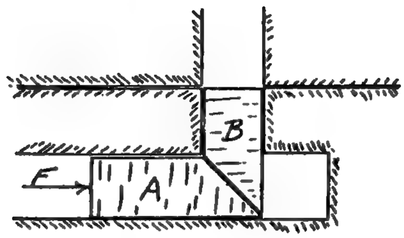
rod 8-1/3 ft to the left of the ring as shown. Weights W are attached to the other end of this cord until the ring just begins to slip. Find

a) the value of W when slipping just begins;

b) the tension in the five foot length of cord, and

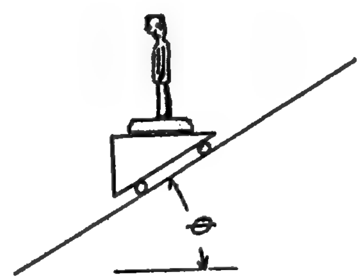
c) the angle θ.

12-5.*

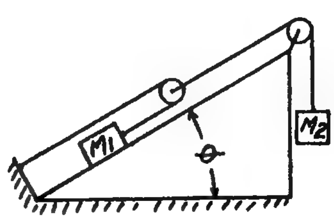



A side view of a simplified form of vertical latch is as shown. The lower member A can be pushed forward in its horizontal channel. The sides of the channels are smooth, but at the interfaces of A and B, which are at $45^{\circ}$ with the horizontal, there exists a static coefficient of friction $\mu$. What is the minimum force F that must be applied horizontally to A to start motion of the latch, if B has a mass m?

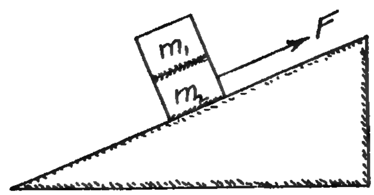
12-6.* A precocious young man performs the following experiment. He arranges a scale with a block of wood and small rollers so that it will roll down an inclined plane without friction, as shown. He stands on the scale, reading it while he moves down the plane. If he normally weighs 160 lb and the scale reading during the descent was 120 lb, what was the inclination angle of the plane?




12-7.* In the system shown, $M_1$ slides without friction on the inclined plane. $\theta = 30^{\circ}$, $M_1 = 400$ g, $M_2 = 200$ g. Find the acceleration of $M_2$ and the tension in the cords.



12-8.*

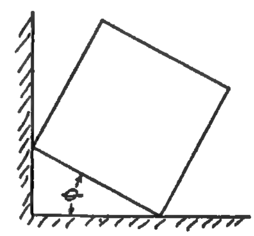



In the arrangement shown, the inclined plane is 130 cm long and its upper end is 50 cm above the level of the lower end. The block $m_2$ rests on the plane, and has a mass of 60 g. The block $m_1$ has a mass 200 g. The coefficient of static friction between the two blocks is 0.50; the coefficient of sliding friction between the lower block and the plane is 0.33. A force F upward and parallel to the plane is applied to the lower block.

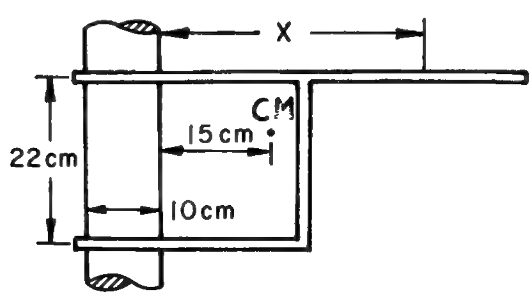
a) What is the acceleration of the lower block when the upper block just starts to slip on it?

b) What is the maximum value of F before this slipping takes place?

12-9.* A cube of mass M rests tilted against the wall as shown in the diagram. There is no friction between the wall and the cube, but the friction between the cube and the floor is just sufficient to keep the cube from slipping. When $0 < \theta < 45^\circ$, find the minimum coefficient of friction as a function of $\theta$. Check whether your answer is reasonable by noting values of $\mu$ for $\theta \to 0$ and $\theta \to 45^\circ$ and by calculating $\theta$ for which $\mu = 1$.



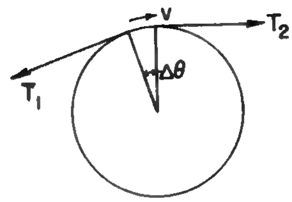

12-10.*




Adjustable supports that can be slid up and down vertical posts are very useful in many applications. Such a support is shown,

with pertinent dimensions. If the coefficient of static friction between post and support is 0.30, and if a load 50 times the weight of the hanger is to be placed on the hanger, what is the minimum value of X for no slipping of the hanger?

12-11.




a) A cord moving at a low speed v rubs against a round post and deviates from a straight line by a small angle $\Delta\theta \ll 1$ radian. If the tension on one side of the post is $T + \Delta T$ and on the other side is T, what is the difference $\Delta T$ introduced by the friction?

b) Integrate the preceding equation to find the ratio of tensions at the two ends of a cord wrapped around a circular post a finite angle $\alpha$ and pulled so as to slip.

12-12. An object rests at the base of a frictionless $20^\circ$ incline 1.00 m long (slant). If the incline is accelerated along the table with an acceleration $a = 4.00$ m sec$^{-2}$, how long does it take the object to slide to the top of the slope?

12-13. What are the dimensions of

a) Electric field,

b) Magnetic induction,

c) Gravitational field,

d) E/B ?

12-14. A charged particle moves in a plane at right angles to a magnetic field $\vec{B}$. Show that the particle moves in a circular path, and find the radius of the circle.

12-15. Find the time required for the particle to go around the orbit of the preceding problem. This result is of importance in the operation of a cyclotron. Why?

12-16. A particle of charge q and mass m moves in a combined electric and magnetic field $E_y$ and $B_z$. (All other field components are zero).

a) Write the equations of motion of the particle.

b) Apply a Galilean transformation to the coordinate system:

$$x' = x - (E_y/B_z)t$$
$$y' = y$$
$$z' = z$$

c) What do you conclude concerning the motion of a particle in crossed electric and magnetic fields?

14-1. A force $\vec{F} = 1.5\, y\, \vec{i} + 3x^2\, \vec{j} - 0.2(x^2 + y^2)\vec{k}$ (Newtons) acts upon a particle of mass 1.00 kg. At t = 0 the particle is located at $\vec{r} = 2\vec{i} + 3\vec{j}$ (meters) and is moving with a velocity $\vec{v} = 2\vec{j} + \vec{k}$ (m sec$^{-1}$). Find, at t = 0

a) The force on the particle,

b) the acceleration of the particle,

c) the kinetic energy, and

d) the rate of change of kinetic energy.

14-2. Approximately what will be the location, the velocity, and the kinetic energy of the above particle at t = 0.01 sec?

Ans: $\vec{r} = 2.00\vec{i} + 3.02\vec{j} + .01\vec{k}$ m

$\vec{v} = .045\vec{i} + 2.12\vec{j} + .97\vec{k}$ m sec$^{-1}$

T = 2.71 J

14-3. The particle of Ex. 14-1 moves from the point (0,-1,0) to the point (0,+1,0) on a frictionless track under the action of the force $\vec{F}$ (plus a certain force of constraint). Find the work done by the force $\vec{F}$ if the track is

a) A straight track along the y-axis

b) A circular track in the z-y plane

Is this a conservative force?

14-4. An object of mass 6.0 kg is free to move along the x-axis on a frictionless track. In each of the cases given it starts from rest at x = 0, t = 0.

a) It moves 3.00 m under the action of a force F = (3 + 4x)N where x is in m

1) What velocity does it acquire?

2) What is its acceleration at that point?

3) What power is being expended on it at that point?

b) It moves for 3.00 sec under the action of a force F = (3 + 4t)N where it is in sec.

1) What velocity does it acquire?

2) What is then its acceleration?

3) What power is being expended on it at that time?

14-5. A spherical shell of radius 0.50 m is (uniformly) charged to a potential of $10^6$ V. Find the charge on the shell.

14-6. Often, a capacitor consists of two (metallic) bodies, equally and oppositely charged. The capacitance C is then defined as the ratio of the charge on one body divided by the potential difference between them:

$$C = Q/(\Phi_2 - \Phi_1) \quad \text{(Farad)}$$

Find the capacitance of a pair of concentric spherical shells, of radii A and B.

14-7. If the Earth carried a net charge of 1.00 C, what would its potential be?

14-8. An automobile weighing 1000 kg is powered by an engine whose rated power is 120 kW. If the engine develops this power at a speed of 60 km $h^{-1}$, what is the maximum acceleration the car can have at this speed?

14-9. A flexible cable of length L which weighs M kg $m^{-1}$ hangs over a pulley of negligible mass, radius, and friction. Initially, the cable is just balanced. It is given a slight push to unbalance it, and it proceeds to accelerate. Find its speed as the end flies off the pulley.

14-10.* Water (density 62.5 lb $ft^{-3}$) is pumped through a smooth hose whose nozzle has a cross-sectional area of 5.5 sq. in. When the nozzle is aimed at an angle of $30^{\circ}$ above the horizontal, the water stream is observed to have the apex of its trajectory 16 ft above the level of the nozzle. The pump inlet is connected to a large reservoir, and the water in the reservoir stands at an elevation 8.0 ft below the nozzle. If the overall efficiency of the pump and the driving motor is 60 percent, what power in kilowatts is being drawn from the electric line feeding the motor?

14-11.* World records (1960) for the shot-put, the discus, and the javelin are respectively 63 ft 4 in, 196 ft $6\frac{1}{2}$ in, and 282 ft $3\frac{1}{2}$ in. The masses of the missiles involved are respectively 16 lb, 4.4 lb, and 1.77 lb. Round off the distances to the nearest foot, and then

compare the work done by each champion in making his record toss, assuming that each trajectory starts at an elevation of 6 ft above level ground and has an initial elevation of $45^{\circ}$. Neglect air resistance.

14-12.* A car with a mass of 3000 lb has a motor of 85 horsepower. To travel at a constant speed of 30 mph on the level the car must use 20 hp. Assuming that the frictional forces are the same, what is the steepest hill the car can climb at this same speed? (Specify the angle or some function of the angle that the slope of the hill makes with the horizontal.)

Ans: $\theta = \sin^{-1} 0.27$

14-13. A certain spherical body of radius R and mass M has a uniform mass density throughout its volume. Find the gravitational potential and the gravitational field strength as a function of the distance from the center. Sketch your results graphically.

14-14. A 25 g weight hanger is attached to a spring of negligible mass whose force constant is $k = 15.3\ \mathrm{N\ m^{-1}}$. A mass $m = 50$ g is dropped from a height $h = 9.0$ cm onto the stationary weight hanger, with which it collides inelastically. What is the minimum height attained by the mass m below its starting point?

14-15. A certain spring has a force constant k. If it is stretched to a new equilibrium length within its linear range, by a constant force F, show that it has the same force constant for displacements from the new equilibrium position.

14-16. A small, frictionless car coasts on an inclined track with a circular loop-the-loop of radius R at its lower end. From what height H above the top of the loop just the car start in order to traverse the loop without leaving the track?

14-17. A particle starts from rest at the top of a frictionless sphere of radius R and slides on the sphere under the force of gravity. How far below its starting point does it get before flying off the sphere?

14-18. A small body of mass m moves under gravity in an elliptical orbit of eccentricity e and semi major axis a about a large mass M. (Assume that M remains stationary.) Evaluate the total energy of m (kinetic plus potential).
(Note that E does not depend upon e.)

14-19. a) Show that the area of an ellipse is $\pi$ ab.
b) Deduce Kepler's third law for elliptical orbits.
c) Show that all orbits having a given total energy per unit mass will have the same period.
(Assume $m \ll M$ for simplicity)

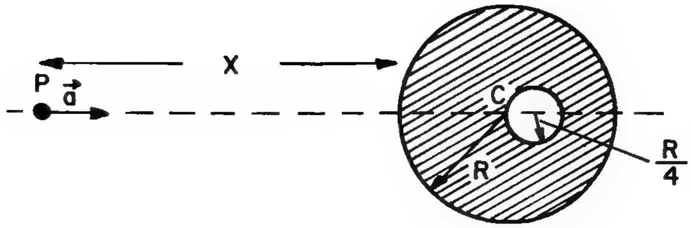
14-20. The speed needed for a body to leave the Earth's gravitational field is (approximately) 7.0 mi sec$^{-1}$. If an interplanetary probe is given an initial speed of 8.0 mi sec$^{-1}$ just above the Earth's atmosphere, with what speed relative to the Earth will it be traveling when it is at a distance of $10^6$ mi from the Earth?

14-21. With what minimum speed must an interstellar probe be launched from near the Earth's surface in order to escape from the solar system with a residual speed of 10 mi sec$^{-1}$ relative to the sun? The speed of the Earth in its orbit is 18.5 mi sec$^{-1}$.

14-22. In the preceding problem, if it is desired to have the probe moving in a prescribed direction when it has escaped from the sun, what then is the maximum launching speed that could be required?

14-23. It is desired to send a solar probe into an orbit with a perihelion distance of 0.010 A.U. and having the same period as the Earth, so that data recorded during the flight may be transmitted to Earth one year after the launching date. With what speed, and in what direction relative to the Earth-sun line, should the probe be launched from the Earth? The orbital speed of the Earth is 30 km sec$^{-1}$.

14-24.




Find the gravitational acceleration $\vec{a}$ at a point P at distance x from the surface of a spherical mass of radius R and density $\rho$, which has a spherical cavity of radius $\frac{R}{4}$ whose center is situated at a distance $\frac{R}{4}$ beyond the center of the large sphere C, on the line PC produced.

# CHAPTER 15

15-1. Solve the Lorentz transformation for x,y,z,t in terms of x', y', z', t'.

15-2. Analyze the operation of the "light-clock" of Fig. 15-3 when it is oriented parallel to the direction of motion. Remember to include the Lorentz contraction.

15-3. A muon is formed high in the atmosphere and travels at a speed $v = 0.990\ c$ for a distance of 5.00 km before it decays.

a) How long does the muon "live," as measured by us and as it would appear in its own frame of reference?

b) What thickness of atmosphere does the muon traverse, as measured in its reference frame?

15-4. The total electrical energy generated in the USA in 1960 amounted to $7.53 \times 10^{11}$ kWh.

a) How much mass was converted into energy in this process?

b) If all of the mass change in the conversion of deuterium into helium were available (some is lost into neutrinos), how much heavy water per second would be needed to supply the necessary deuterium?

NOTE: $M_{H^2} = 2.0147$ amu $M_{He^4} = 4.0039$ amu

15-5. The total power incident at the top of the Earth's atmosphere from the sun is about 1.4 kW $m^{-2}$. If this energy all arises

from the conversion of ordinary hydrogen into helium, how much hydrogen, in metric tons per second, does the sun "burn"? (Neglect the loss into neutrinos.)

15-6. A particle of rest mass $m_o$ is caused to move along a line in such a way that its position is

$$x = \sqrt{b^2 + c^2t^2} - b$$

What force must be applied to the particle to produce this motion?

15-7. a) Evaluate the acceleration of gravity in l.y. $y^{-2}$.

b) If an isolated space ship accelerates at such a rate that its occupants feel a constant acceleration equal to that of gravity at the Earth's surface, and does so for a period of 5.00 y as measured by a stationary (unaccelerated) observer who is at rest with respect to the ship at $t = 0$, how far has the ship gone, and how fast is it traveling, at the end of this time?

# CHAPTER 16

16-1. Write the Lorentz transformation in differential form, $dx = \gamma(dx' + \beta c dt')$ etc., and thus evaluate $dx/dt = v_x$ in terms of $v'_x$, $V$, etc.; do the same for $dy/dt = v_y$.

16-2. A particle which moves along the x-axis has a velocity $v_x$ and an acceleration $a_x$. What velocity and acceleration will it appear to have in the S' system which is moving at velocity V with respect to the first system?

16-3. Carry through the check of the formula $m_w = m_v\sqrt{1 - u^2/c^2}$ as outlined at the end of p. 16-7.

16-4. A particle of rest mass $m_o$, moving at speed $v = 4c/5$, collides inelastically with a similar particle at rest.

a) What is the speed of the composite particle?

b) What is its rest mass?

16-5. The Berkeley "bevatron" was designed to accelerate protons to sufficiently high energy to produce proton-anti-proton pairs by the reaction

$$p + p \longrightarrow p + p + (p + \bar{p})$$

The so-called threshold energy of this reaction corresponds to the situation when the four particles on the right move along together as a single particle of rest mass $M = 4\ m_p$. If the target proton is at rest before the collision, what kinetic energy must be bombarding proton have at threshold?

17-1. The rest mass of a proton is $m_p = 938$ MeV. In the cosmic radiation, protons having an energy of about $10^{10}$ GeV have been detected by indirect methods. Assume that a proton of this energy travels diametrically across a galaxy whose diameter is about $10^5$ l.y. How long does this require, as measured in the proton's reference frame?

17-2. Show that an electron has a rest energy $m_e c^2 = 0.511$ MeV.

17-3. A pion ($m_\pi = 273\ m_e$) at rest decays into a muon ($m_\mu = 207\ m_e$) and a neutrino ($m_\nu = 0$). Find the kinetic energy and momentum of the muon and the neutrino in MeV.

17-4. If q is measured in units of an electron charge, p is measured in MeV, and B is measured in gauss, what is the relation between p, B, and R? Let $q = Zq_e$.

17-5. A cyclotron is being designed to accelerate protons to a kinetic energy of 150 MeV. The magnetic field strength is to be $1.00 \times 10^4$ G.

a) What must be the radius of the magnet pole piece?

b) What frequency must be used on the acceleration electrodes?

c) By what percentage must the driving frequency be changed to allow for relativistic effects during the acceleration of a given particle?

18-1. A force $\vec{F} = 30\,\vec{i} + 40\,\vec{j}$ Newtons acts on the point at $\vec{r} = 8\,\vec{i} + 6\,\vec{j}$ meters. Find:

a) The torque about the origin

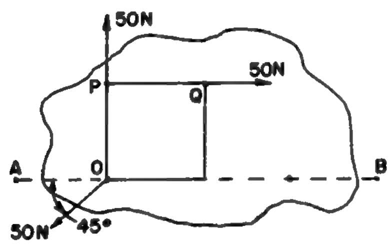
b) The magnitude of the lever arm of the force

c) The component of the force perpendicular to $\vec{r}$.

18-2. At what latitude is the tangential speed of a point due to the earth's rotation 200 m $s^{-1}$ less than it is in Pasadena?

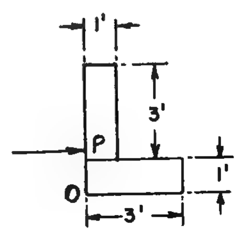
Ans: $\lambda' = 66.6^o$

18-3.

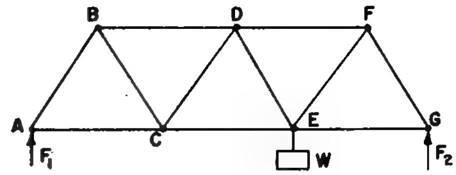



A flat steel plate floating on mercury is acted upon by three forces at three corners of a square of side 0.100 m, as shown. Find a single fourth force which will hold the plate in equilibrium. Give the magnitude, direction, and point of application along the line AB.

18-4. The L-shaped body shown in the figure is made of sheet metal of uniform thickness and rests on a smooth, horizontal table. It is struck with a sudden blow in the direction shown, and is observed to move away without rotating. How far from the vertex O was the blow applied?



18-5. A bridge truss is constructed as shown, all joints being frictionless pivots and all members rigid, weightless, and of equal length. Find the reaction forces $F_1$ and $F_2$ and the force in the member DF.




18-6. Calculate the moments of inertia of the following rigid bodies, each of which has a mass m:

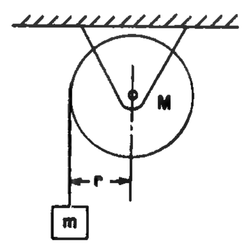
a) a thin, straight uniform rod of length L, about a perpendicular axis through one end,

b) a thin, straight, uniform rod of length L, about a perpendicular axis through its center,

c) a thin-walled hollow circular cylinder of radius r, about its axis, and

d) a solid circular cylinder of radius r, about its axis.

18-7. A mass m is hung from a string wound around a solid circular cylinder of mass M and radius r, pivoted on frictionless bearings as shown. Find the acceleration of m.




18-8. A mass m moves on the surface of a smooth horizontal table, guided by a string attached to m and passing downward through a small hole in the table top. Initially the length of string above the table

is $r_1$, and the mass is set moving in a circular path of this radius, at speed $v_1$. The string is then pulled downward through the hole until an amount $r_2$ remains above the table. Find:

a) the final speed of the mass m,

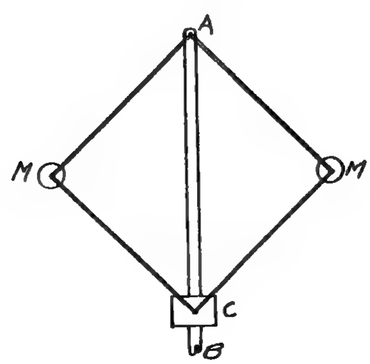
b) the work required to pull the string through the hole from $r_1$ to $r_2$, and

c) the magnitude of the force needed to hold the radius constant, using the principle of virtual work.

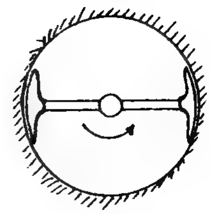
18-9. Find the angular momentum of a planet of mass m moving in a circular orbit of radius R. Using this result, deduce that the distance of the moon from the earth will increase over a long period of time because of its tidal drag on the earth's rotation. Also, discuss the conservation of energy in the earth-moon system.

18-10. Solve Ex. 4-9, Chapter 4, using the conditions that the net force and the net torque acting on a body in static equilibrium are zero.

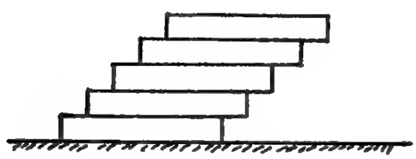
18-11.* A rotating governor, as shown, is to be designed to shut off power when the machine to which the governor is directly connected reaches a speed of 120 rpm. The operating collar C weighs 10.0 lb and slides without friction on the vertical shaft AB. C is so designed to shut off power when the distance AC reduces to 1.41 ft. If the four links of the governor framework are each 1.00 ft long between frictionless pivots and are relatively massless, what value should the masses M have so that the governor will operate as planned?



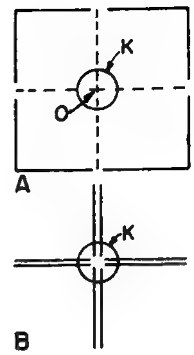
18-12.* The essential elements of one form of simple speed governor are as shown: to a vertical shaft a horizontal rod is mounted symmetrically and on the horizontal rod are freely sliding brake shoes. When the shaft turns, the brake shoes press against the inner surface of a stationary cylindrical brake drum. If the brake shoes are each of mass m, and their thickness dimension is negligible compared to the inner radius of the brake drum r, if the coefficient of sliding friction between the shoes and the drum is $\mu$, develop the formula for the power required to turn the governor shaft in terms of m, r, $\mu$, and f, the frequency of rotation of the shaft.



18-13.* A uniform brick of length L is laid on a smooth horizontal surface. Other equal bricks are now piled on as shown, so that the sides form continuous planes, but the ends are offset at each brick from the previous brick by a distance $\frac{L}{a}$, where a is an integer. How many bricks can be used in this manner before the pile topples over?



19-1.* Eight thin uniform rods, each of length L and mass m, are held in the form of a plane square by the massless framework shown dotted in Fig. 19-1A. The square is set rotating freely about a frictionless axle through O, perpendicular to the plane of the framework, with an angular speed of $\omega_o$ rad $\sec^{-1}$. While thus rotating an internal mechanism K attached to the framework and with an unchanging moment of inertia about O of $\frac{40}{3}$ $ML^2$, collapses the square to the cross shown in Fig. 19-1B. How much work was done by the mechanism in the collapsing process?




19-2. The elastic restoring torque exerted by a torsion fiber is proportional to the angle of twist: $\tau_{fiber} = - k\Theta$.

a) Show that the potential energy of such a fiber twisted through an angle $\Theta$ is $U = \frac{1}{2}k\Theta^2$.

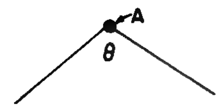
b) The deflecting torque exerted on a galvanometer coil is given by the expression

$$\tau = n\,AB\,i$$

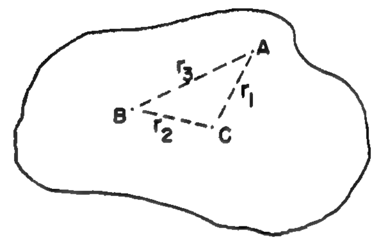
where $i$ = current through the coil
$n$ = number of turns of wire on the coil
$B$ = the magnetic field produced by the permanent galvanometer magnet

In the laboratory experiment on the speed of a bullet, the charge on a capacitor is measured by discharging the capacitor through a galvanometer coil and noting the resulting maximum deflection. Here $|i| = |dq/dt|$, and the discharge takes place so quickly that the galvanometer coil does not appreciably move away from its initial $\Theta = 0$ position during the time that current flows. Neglecting friction, show that the maximum "throw" of the galvanometer is proportional to the initial charge on the capacitor.

19-3.* A straight, uniform wire of length L and mass M is bent at its midpoint to form the angle $\Theta$. What is its moment of inertia for an axis passing through the point A, perpendicular to the plane determined by the bent wire?

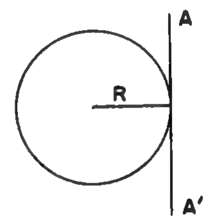



19-4. An irregular plate of metal of uniform thickness and mass M has its center of mass at C, and the moment of inertia for an axis perpendicular to the sheet is known, $I_a$. Under what conditions of $r_1$, $r_2$, and $r_3$ can one correctly express the moment of inertia for an axis through B, also perpendicular to the sheet, as




$$I_b = I_a + Mr_3^2 \text{ ?}$$

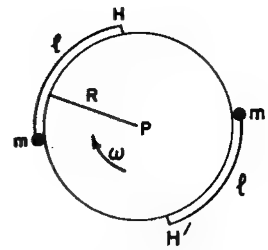
19-5. A circle of radius R is revolved around an axis AA' tangent to it to generate a torus. Find the volume of this torus.




19-6. Masses $M_1$ and $M_2$ are placed at the opposite ends of a rigid rod of length L and negligible mass; the dimensions of $M_1$ and $M_2$ are negligible compared to L. The rod is to be set rotating about an axis perpendicular to it. Through what point on this rod should this axis pass in order that the work required to set the rod rotating with an angular speed $\omega_o$ shall be a minimum?

19-7. A uniform circular disk of radius R and mass M is arranged to spin freely with angular speed $\omega$ on a horizontal plane on a pivot P at

its center. Pinned to its edge are two small masses m attached by cords of length l wrapped around its periphery, as shown. While the disk is spinning, these masses are released simultaneously without disturbing the angular momentum of the system. Thereupon, the small masses fly off, their restraining cords being released from hooks H,H' when the cords extend radially outward. Find $\ell$, the length of these cords such that the disk will be stopped by the action.

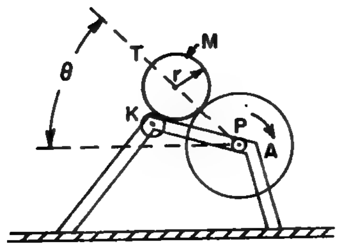



Note: This scheme has been used to reduce the spinning motion of satellite vehicles.

19-8. If Moe (coordinates x', y') rotates relative to Joe (coordinates x,y), who is at rest, find the equations for the apparent force components that must act on a particle according to Moe and show that these consist of the components of the true force $\vec{F}$ as seen by Joe, plus two pseudo-forces: a radial centripetal force and a Coriolis force at right angles to the velocity.

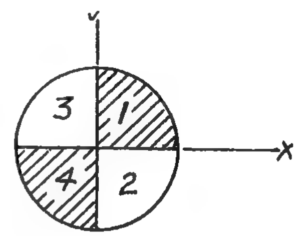
19-9. A certain uniform bowling ball of radius R and mass M is initially launched so that it is sliding with speed $V_0$ without rolling on an alley with a coefficient of friction $\mu$. How far does the ball go before it starts rolling without slipping, and what is then its speed?

19-10.

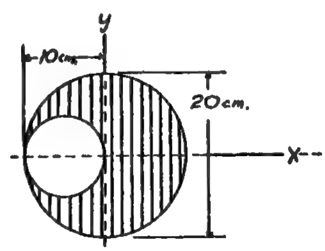



An adaptation of an inking arrangement for a printing press is as shown in the figure. K is a firmly supported, but idling, inking roller of negligible moment of inertia; P is a driven press roll firmly supported and T is a transfer roll freely floating between K and P. T is a solid cylinder of radius r and mass M; it always rolls without slipping on both K and P, and the geometry is such that the line of centers TP is $\Theta$ above the horizontal. What is the maximum angular acceleration A that can be given to P without T losing contact with K?

19-11.* A solid cylinder has a density which varies by quadrants as shown, with the numbers indicating relative densities. If the x-y axes are as indicated, what is the equation of the line drawn through the origin and through the center of mass?




19-12.* A disc of uniform density has a hole cut out of it, as shown. Find the center of mass.




19-13. Find the CM of a thin uniform wire of length L, bent into a circular arc of radius R ($R > L/2\pi$). Use coordinates with origin

at the center of the circle and with the x-axis passing through the center of the wire.

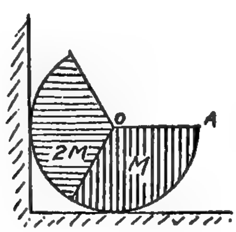
19-14. From the result of the preceding exercise, or otherwise, find the CM of a sector of a uniform disc of radius R which subtends an angle $\alpha$ at the center.

19-15.* A cylinder of radius $\pi$ cm and mass 3 kg is cut into thirds. The same thing is done to a second cylinder of radius $\pi$ cm and mass 6 kg. A piece from one cylinder is glued to a piece from the other one giving the arrangement shown, where the radius OA is horizontal. The floor is perfectly rough, and the wall is perfectly smooth.

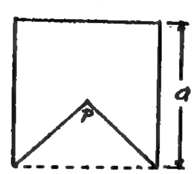
a) What is the force of the cylinder on the wall?

b) How far from the center along the radius OA would one have to place a point mass M so that the system would remain in equilibrium if the wall were removed?

Note: To get the location of the CM of a sector of a circle, one can integrate, or look it up in a handbook.

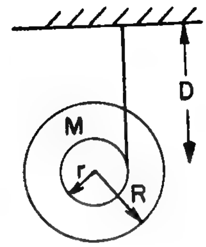



19-16.* From a square piece of uniform sheet metal an isosceles triangle is to be cut out from one edge, as shown, such that the remaining



metal, when suspended from the apex P of the cut, will remain in equilibrium in any position. What is the altitude of the cutout triangle?

19-17. A yo-yo like spool consists of two uniform discs, each of mass M and radius R, and an axle of radius r and negligible mass. A thread wound around the axle is attached to the ceiling, and the spool is released from rest a distance D below the ceiling.




a) If there is to be no pendulum -- like swinging motion, what angle should the thread make with the vertical as the spool is released?

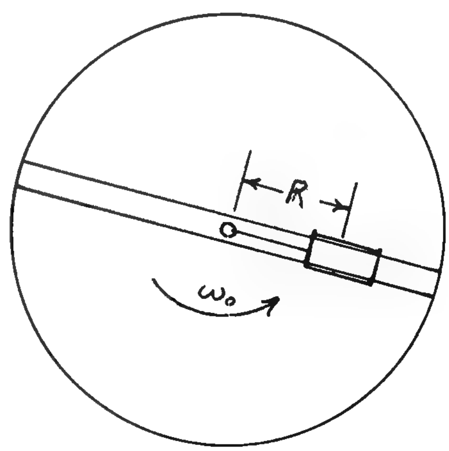
b) What is the downward acceleration of the center of the spool?

19-18. A turntable of moment of inertia $I_o$ rotates freely on a hollow vertical axis. A cart of mass m runs without friction on a straight radial track on the turntable. A cord attached to the cart passes over a small pulley and then downward through the hollow axis. Initially the entire system is rotating at angular speed $\omega_o$, and the cart is at a fixed radius R from the axis. The cart is then pulled inward by applying an excess force to the cord, and eventually arrives at radius r, where it is allowed to remain.

a) What is the new angular velocity of the system?

b) Show in detail that the difference in the energy of the system between the two conditions is equal to the work done by the centripetal force.

c) If the cord is released, with what radial speed $\dot{r}$ will the cart pass the radius R?



# CHAPTER 20

20-1. By writing the vectors in component form, or otherwise, prove the five relations involving vector products given in the Summary.

20-2. A rigid body is rotating with an angular velocity $\vec{\omega}$ about a fixed axis. Show that the velocity of any point P in the body is $\vec{v} = \vec{\omega} \times \vec{r}$, where $\vec{r}$ is a vector from any point on the axis of rotation to the point P.

20-3. A rigid body is rotated through an infinitesimal angle $\Delta\theta_1$ about a certain axis and is then rotated through an infinitesimal angle $\Delta\theta_2$ about some other axis intersecting the first axis at some point O. Show that the net displacement of any point in the body is the same as it would be if it were instead rotated through a single infinitesimal angle about some intermediate axis, and show how to find this axis and angle. Use this to prove that a rigid body subjected simultaneously to angular velocities about various axes moves as it would with a single angular velocity equal to their vector sum, treating each angular velocity as a vector of length $\omega$ directed along the axis of rotation.

20-4. A collection of N particles with masses $m_i$, positions $\vec{r}_i$, and velocities $\vec{v}_i$ have a certain angular momentum

$$\vec{L} = \sum \vec{r}_i \times \vec{p}_i = \sum m_i \vec{r}_i \times \vec{v}_i$$

On the other hand, as viewed in a coordinate system moving with their center of mass, suppose they have an angular momentum $\vec{L}_{CM}$. If $\vec{R}_{CM}$ and $\vec{V}_{CM}$ are the position and velocity of the CM, and $M = \sum m_i$ is the total mass of the particles, show that

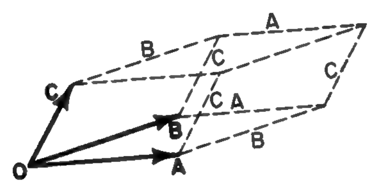
$$\vec{L} = \vec{L}_{CM} + M\,\vec{R}_{CM} \times \vec{V}_{CM}$$

Compare this result with that of Ex. 11-9.

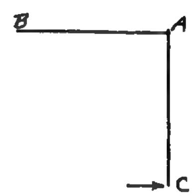
20-5. a) Any three vectors A, B, and C may be thought of as defining a solid body having six faces, parallel in pairs -- a parallelepiped. Show that the volume enclosed by such a figure is

$$V = |\vec{A} \cdot (\vec{B} \times \vec{C})|$$

b) A parallelepiped with one vertex at the origin has three adjacent vertices at the points (10,-5,3), (3,-4,7), and (-5,-6,3) in rectangular coordinates (x,y,z). What is its volume?

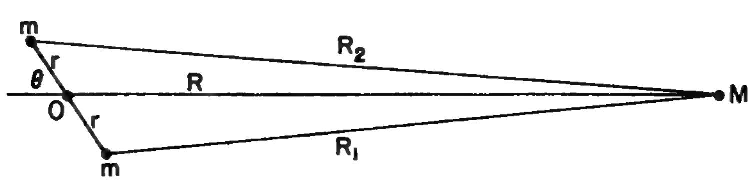



20-6. Two uniform equal stiff rods AB and AC are freely hinged at A and placed on a smooth horizontal table with AC ⊥ AB. A horizontal blow is delivered perpendicular to AC at C. Find the ratio of the resulting linear velocities of the centers of mass of AB and of AC, immediately after impact.




20-7. A flywheel having the shape of a uniform thin circular plate of mass 10.0 kg and radius 1.00 m is mounted on a shaft passing through its CM but making an angle of $1^{\circ}0'$ with its plane. If it rotates about this axis with angular velocity 25.0 radians $\sec^{-1}$, what torque must be supplied by the bearings?

20-8. Two equal masses m are fixed on a massless rod a distance 2r apart, and are attracted gravitationally by a mass M, situated at a distance R $\gg$ r from the center O of the rod. The rod makes an



angle $\theta$ with R. Find the approximate value of the torque on the rod about its center.

20-9. The moon and the sun both exert a torque upon the earth because of the earth's oblateness. Which body exerts the greater torque, and by approximately what factor? (See preceding exercise.)

20-10. The equatorial and polar radii of the earth are 6378.388 km and 6356.912 km. Its specific gravity $\rho$ at various depths D below the surface are shown in the Table below (* denotes a discontinuity).

| D(km) | $\rho$ |
|---|---|
| 0 | 2.60 |
| 30* | 3.0<br>3.3 |
| 100 | 3.4 |
| 200 | 3.5 |
| 400 | 3.6 |
| 1000 | 4.7 |
| 2000 | 5.2 |
| 2900* | 5.7<br>9.4 |
| 3500 | 10.2 |
| 5000* | 11.5<br>16.8 |
| 6000 | 17.1 |

Using these values, estimate:

a) the moment of inertia of the earth,

b) its rotational angular momentum,

c) its rotational kinetic energy, and

d) the time required for the rotational axis to precess about the pole of the ecliptic due to the torques of the moon and the sun.

(The tilt of the earth's axis is $23\frac{1}{2}^{\circ}$.)

20-11. a) Starting from rest, a symmetrical object rolls (without slipping) down an incline of height h. The moment of inertia of the object about its center of mass is I, the mass is M, and the radius of the rolling surface in contact with the incline is r. Determine the linear velocity of the center of mass at the bottom of the incline.

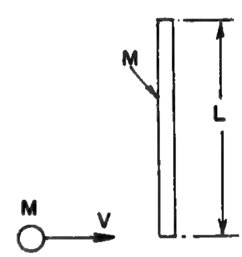
b) Apply the general equation of a) to determine the velocity of the center of mass if the object is

i) a sphere

ii) a disc

iii) a disc of mass $M_1$ outer radius $R_1$ with a spindle of mass $m_2$ and radius $r_2$ on which the disc rolls.

20-12. A thin rod of mass M and length L rests on a horizontal frictionless surface. A small piece of putty, also of mass M, and with velocity v directed perpendicularly to the rod, strikes one end and sticks, making an inelastic collision of very short duration.




a) What is the velocity of the center of mass of the system before and after the collision?

b) What is the angular momentum of the center of mass of the system about its center of mass just before the collision?

c) What is the angular velocity (about the center of mass) just after the collision?

d) How much kinetic energy is lost in the collision?

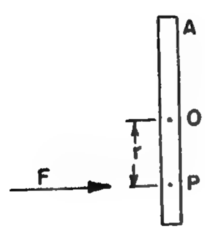
20-13. If all the ice on earth were to melt, the height of mean sea level would increase by about 200 ft. Taking the mean latitude of the existing ice caps as $80^{\circ}$, and neglecting the irregular distribution of the oceans, by about how many seconds would the length of the day increase? Assume the earth is a sphere of radius 6370 km and moment of inertia $8.11 \times 10^{37}$ kg $m^2$.

20-14. A uniform rod of length L and mass M is at rest on a frictionless horizontal surface. The rod receives an impulse $J = \int F dt$ of very short duration applied at right angles to the rod at a point P where OP = r.

a) Just after the impulse, what is the velocity of the center of mass O? What is the angular velocity about O? What is the instantaneous velocity of the end point A?

b) Determine the distance AP for which the velocity of the point A is zero just after the impact.

c) If the rod is supported vertically from a pivot at A, where should a blow be struck to set the rod in rotation about A without exerting an initial sidewise force on the pivot?



# CHAPTER 21

21-1. A certain rigid body of mass M is supported on a frictionless horizontal axis which lies a distance <u>d</u> from the CM. The moment of inertia about the axis of rotation is I.

a) Write the differential equation which describes the variation of the angle $\Theta$ with time, where $\Theta$ is measured from the equilibrium position of the body.

b) If the body undergoes small oscillations, so that $\sin \Theta \approx \Theta$, what is their period?

21-2. In the preceding exercise, the moment of inertia of the rigid body about its CM is $I_c$. Find an expression for the period of small oscillations as a function of d (and $I_c$), and thus show

a) that there are two values of d, say $d_1$ and $d_2$, which correspond to a given period,

b) that the period is $t_o = 2\pi \;\; \dfrac{d_1 + d_2}{g}$ in terms of $d_1$ and $d_2$,

c) that the period has a minimum value when $d = \sqrt{I_c/M}$ . Find this minimum period.

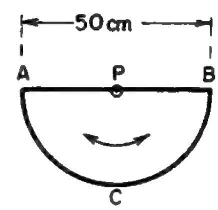
21-3. A certain linear spring has a free length D. When a mass <u>m</u> is hung on the end, it has a length D + A. While it is hanging motionless with mass <u>m</u> attached, a second mass <u>m</u> is dropped from a height A onto the first one, with which it collides inelastically. Find the period, amplitude, and the maximum height (above the original equilibrium position) attained in the resulting motion.

21-4. Two particles A and B execute harmonic motion of the same amplitude (10 cm) on the same straight line. For particle A, $\omega_A = 20\ \text{sec}^{-1}$; for B, $\omega_B = 21\ \text{sec}^{-1}$. If at $t = 0$, they both pass through $x = 0$ in the positive x-direction (hence are then "in phase"),

a) How far apart will they be at $t = 0.350$ sec?

b) What is the velocity of B relative to A at 0.350 sec?

21-5. A frame made of stiff wire of uniform cross section and density consists of a semi-circular arc ACB with its diameter AB. It is hung from a frictionless pin P passing through a hole at the midpoint of its diameter, and is set into vibration as a pendulum in its own plane. If the diameter of the frame AB is 50 cm, what is the period of the oscillating motion for small arcs?




21-6. A 20 g weight hanger with a 5 g weight on it is hung from a vertical spring of negligible mass. When the spring is displaced from equilibrium the system is found to vibrate in vertical S.H.M. with a period of $\pi/3$ sec. If the 5 g weight is replaced by a 25 g weight, how far can the spring be displaced from equilibrium before release if the weight is not to jump off the weight hanger?

21-7. Two particles, of mass $\frac{3M}{4}$ and M, are connected by a massless spring of free length L and force constant k. These masses are initially at rest L apart on a horizontal frictionless table. A particle of mass $\frac{M}{4}$, moving with speed v along the line joining the two connected masses, collides with and sticks to the particle of mass

$\frac{3M}{4}$ . Find the amplitude and period with which the spring between the two masses vibrates.

21-8. The gravitational force felt by a particle embedded in a solid uniform sphere, due to the mass of the sphere only, is directly proportional to the distance of the particle from the center of the sphere. If the earth were such a sphere, with a narrow hole drilled through it along a polar diameter, how long would it take a body dropped in the hole to reach the surface at the opposite side of the earth?

21-9. In its initial stages, a colony of bacteria grows at a rate proportional to the number of bacteria present. Write the differential equation which expresses this relationship.

21-10. The pivot point of a simple pendulum having a natural period of 1.00 sec is moved laterally in a sinusoidal motion with an amplitude 1.00 cm and period 1.10 sec. With what amplitude should the pendulum bob swing after a steady motion is attained?

## CHAPTER 22

The most general kind of number which satisfies the rules of elementary algebra is a complex number. Complex numbers may be written as a sum of a pure real (positive or negative) number and a pure imaginary number.

$$(\text{complex number})\ u = (\text{real number})\ x + (\text{imaginary number})iy$$

$i = \sqrt{-1}$ is called the unit imaginary number

$1 = \sqrt{+1}$ is called the unit real number

Any algebraic equation is still true if the sign of i is changed throughout. This is called taking the complex conjugate. If $u = x + iy$, then the complex conjugate of u, written $u^*$, is $u^* = x - iy$.

The rules of algebra, applied to complex numbers, show that

$$(a + ib) + (c + id) = (a + c) + i(b + d)$$

$$(a + ib)(c + id) = (ac - bd) + i(ad + bc)$$

$u = \sqrt{uu^*} = \sqrt{x^2 + y^2}$ is called the magnitude of u.

A real number raised to an imaginary power is complex, and has unit magnitude. The real and imaginary parts oscillate like the sine and cosine as the magnitude of the imaginary power increases. Specifically,

$$e^{i\Theta} = \cos\Theta + i\sin\Theta$$

---

22-1. In the equation

$$u + iv = (a + ib)\ (c + id)$$

let $b/a = \tan\alpha$ and $d/c = \tan\beta$. Using Eq. 22.4 and formulas of trigonometry, show that

a) $\sqrt{u^2 + v^2} = \sqrt{a^2 + b^2}\ \sqrt{c^2 + d^2}$

b) $v/u = \tan(\alpha + \beta)$

22-2. Work the above exercise using Eq. 22.9.

22-3. Show that $\cos\Theta = (e^{i\Theta} + e^{-i\Theta})/2$

$$\sin\Theta = (e^{i\Theta} - e^{-i\Theta})/2i$$

22-4. Show that

$$(a + ib)/(c + id) = [ac + db + i(bc - ad)]/(c^2 + d^2)$$

22-5. The quantities $\cosh\Theta$ and $\sinh\Theta$, defined as

$$\cosh\Theta = (e^{\Theta} + e^{-\Theta})/2$$

$$\sinh\Theta = (e^{\Theta} - e^{-\Theta})/2$$

are called the hyperbolic cosine and hyperbolic sine of $\Theta$. Show that

$$\cos i\Theta = \cosh\Theta$$

$$\sin i\Theta = i\sinh\Theta$$

and $\cosh^2\Theta - \sinh^2\Theta = 1$

22-6. Using the fundamental formula of differentiation

$$df/dx = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Show that

$$d(e^{\alpha x})/dx = \alpha e^{\alpha x}$$

22-7. a) By successive differentiation, or otherwise, show that $e^x$ may be represented by the infinite series

$$e^x = 1 + x + x^2/2! + x^3/3! + \ldots$$

b) Show that $\cos x$ and $\sin x$ may be represented by the infinite series

$$\cos x = 1 - x^2/2! + x^4/4! + \ldots$$

$$\sin x = x - x^3/3! + x^5/5! + \ldots$$

(These series are of considerable value in calculating $e^x$, $\cos x$, and $\sin x$ for $x \ll 1$, although they converge for all $x$.)

22-8. Find the complete algebraic solution of the equation

$$y = \sqrt[n]{1}$$

where n is an integer.

22-9. Using the properties of $e^{in\Theta}$ and the binomial theorem, show that

$$\cos n\Theta = \cos^n \Theta - \frac{n(n - 1)}{2!} \times$$
$$\cos^{n-2} \Theta \sin^2 \Theta + \ldots$$

22-10. a) From the relation $e^{i(\Theta + \Phi)} = e^{i\Theta} \cdot e^{i\Phi}$, prove the trigonometric formulas giving the cosine and sine of the sum of two angles.

b) Interpret geometrically the result of multiplying one complex number, $Ae^{i\Theta}$, by another complex number, $Be^{i\Phi}$.

22-11. From the following table of successive square roots of 11, find (to 3 places) $\log_{11}2$; $\log_{11}7$

| Root r | $\sqrt[r]{N}$ | $\log \sqrt[r]{N}$ |
|---|---|---|
| 1 | 11.000 | 1.00000 |
| 2 | 3.3167 | 0.50000 |
| 4 | 1.8212 | 0.25000 |
| 8 | 1.3495 | 0.12500 |
| 16 | 1.1617 | 0.06667 |
| 32 | 1.0778 | 0.03333 |
| 64 | 1.0382 | 0.01667 |
| 128 | 1.0195 | 0.00833 |

(Check your result by $\log_a N = \log_a b \log_b N$ where $\underline{a}$ and $\underline{b}$ are any two bases.)

# CHAPTER 23

23-1. Write down and solve the differential equations which describe the steady-state current which flows when a sinusoidal voltage of angular frequency $\omega$ is applied to

a) an inductance L; and

b) a capacitance C.

Thus find the (complex) impedance of an inductance L and a capacitance C.

23-2. Find the impedance $\hat{Z}$ of an inductance L and a capacitance C as a function of $\omega$ when they are connected

a) in series, and

b) in parallel.

Discuss your answers qualitatively.

23-3. a) Show that the differential equation of motion of a mass m on a spring whose force constant is k, and having a frictional force $-m\gamma v$ is

$$d^2x/dt^2 + \gamma dx/dt + \omega_o^2 x = 0$$

where $\omega_o^2 = k/m$.

b) Solve this equation (using complex variables) by assuming a solution of the form $x = e^{\alpha t}$ and thus show that the general solution is

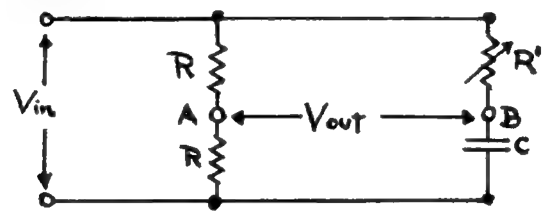
$$x = e^{-\frac{1}{2}\gamma t}(A \cos \sqrt{\omega_o^2 - \gamma^2/4}\ t + B \sin \sqrt{\omega_o^2 - \gamma^2/4}\ t)$$

if $\gamma < 2\omega_o$

c) What is the solution if $\gamma > 2\omega_o$?

23-4. At $t = 0$, the position and velocity of the mass m of the preceding exercise are $x = x_o$ and $\dot{x} = v_o$. Find A and B.

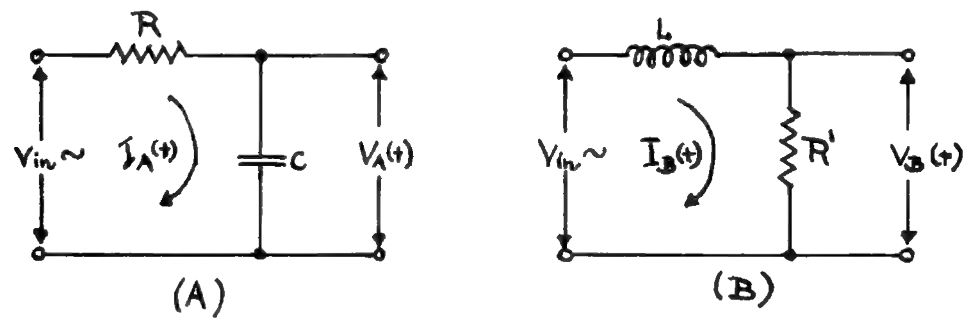
23-5. In electronic circuits it is often desired to provide a sinusoidal voltage of constant amplitude but variable phase. A circuit which accomplishes this is called a <u>phase-shifting network</u>. One example of such a network is shown in the figure. Show that the voltage measured between terminals A and B has half the amplitude of the input voltage, and a phase which may be adjusted between $0^\circ$ and $180^\circ$ by changing the resistance R'.




23-6. The input terminals of circuits A and B (see figure) are connected to an input source of voltage $V_{in} = V_o \cos \omega t$. Assume that the current flow into the output terminals is negligible.

a) Find what relation must exist between R, C, R', and L in order that the steady-state output voltages $V_A(t)$ and $V_B(t)$ may be equal.

b) Find the steady state currents $I_A(t)$ and $I_B(t)$.



24-1. The gliders used with the linear air trough lose speed mainly because of the frictional drag of the thin air film which supports them. This viscous force is proportional to the velocity. Write and solve the differential equation of motion of a glider on a level track. How should the velocity vary with

a) time, and

b) distance?

24-2. A capacitor of capacitance C is initially charged to a voltage $V_o$, and at $t = 0$ it is connected across a resistor of resistance R. Write the differential equation for V as a function of t, and solve it by assuming an exponential solution.

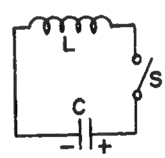
24-3. A certain glider in a tilted air trough has a magnet embedded in it, and this magnet generates eddy currents in the trough which react back on the magnet, giving a retarding force precisely proportional to the velocity. If the glider starts from rest, find (as a function of the angle of tilt of the track, and the drag coefficient $\gamma$ of the magnet)

a) the terminal velocity attained,

b) the velocity as a function of time,

c) the position as a function of time.

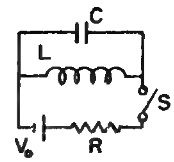
24-4. A capacitance C and inductance L are connected together as shown; the capacitor is initially charged to a voltage $V_o$ and the switch S is open. At time $t = 0$, S is closed.




a) Find the voltage on C as a function of the time.

b) Calculate the quantities $CV^2/2$ and $LI^2/2$ as functions of the time. What do you believe is the significance of these quantities?

24-5. In the circuit shown above, the switch S is initially closed, and a steady current $I = V_o/R$ is flowing. At $t = 0$, S is suddenly opened. Devise a method for finding the maximum voltage that is subsequently observed on the capacitor.




24-6. An object of mass 5.0 kg is found to oscillate with negligible damping when suspended from a spring which causes it to perform 10 complete cycles in 10.0 sec. Thereafter, a certain small magnetic damping is applied, strictly proportional to the velocity of motion, and the amplitude decreases from 0.20 m to 0.10 m in 10 cycles.

a) Write the equation of the motion, with the coefficients of $d^2x/dt^2$, $dx/dt$ and $x$ represented by <u>numbers</u> in MKS units.

b) What is now the period of the motion?

c) In how many cycles (starting from 0.20 m amplitude) will the amplitude reach 0.05 m? 0.02 m?

d) What is the maximum rate of dissipating energy by damping during the first cycle?

24-7. Damped harmonic oscillator: A mass m is suspended from a spring of elastic constant k in a medium which exerts a damping force $-m\gamma \frac{dx}{dt}$.

a) For the case of under damped motion find the complete solutions for the position $x = x(t)$ of m for all times $t \geq 0$ for the following driving forces:

1) $$F = \begin{cases} 0 \text{ for } t < 0 \\ F_o = \text{const for } t \geq 0 \end{cases}$$

2) no driving force, but at $t = 0$ an impulse $J = J_x$ is imparted to mass m.

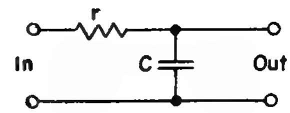
3) $$F = \begin{cases} 0 \text{ for } t < 0 \\ F_o \cos \omega_o t \quad \text{for } t \geq 0 \end{cases} \qquad \omega_o = \sqrt{\frac{k}{m}}$$

b) If the oscillator is driven by a sinusoidal force $F = F_o \cos \omega t$ and we consider long times, what is the frequency $\omega^*$ for which the amplitude reaches a maximum?

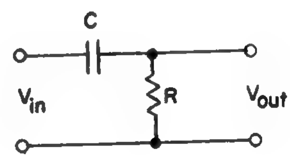
Note: Remember that the complete solution contains both steady-state and transient motion and that the initial conditions determine the constants of integration.

# CHAPTER 25

25-1. In order to suppress the 120 cycle sec$^{-1}$ "hum" from the power supply rectifier of an amplifier, a "smoothing filter" is used. In its simplest form, this consists of a resistance in series with a capacitance, as shown in the figure. If the applied voltage has a DC component $V_0$ and a 120 cycle component of amplitude $V_2$, find the corresponding voltages at the terminals of the capacitor, for $r = 10^3\Omega$ and $C = 10\mu F$.




25-2. In many instances it is desirable to have an electronic circuit which will "differentiate" a function with respect to time. A simple circuit to accomplish this is shown in the figure. Show that the output voltage of this circuit (if negligible current is allowed to flow into the output circuitry) is



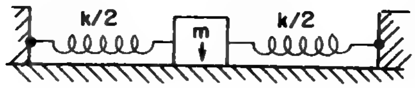

$$V_{out}(t) = RC\,\frac{dV_{in}}{dt}$$

provided $|V_{out}| \ll |V_{in}|$

25-3. Solve for $V_{out}$ in the above circuit for the case in which $V_{in} = V_0 \cos \omega t$, and thus test the validity of the result of the preceding exercise as a function of $\omega$.

25-4. Invent a simple circuit which will "integrate" a function, and discuss its properties.

25-5.

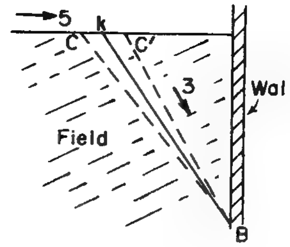



A mass m, attached to two equal horizontal springs of force constant $k/2$, slides on a table top whose coefficient of friction is $\mu$, assumed constant. The mass is pulled aside a distance A from the center point and is then released from rest.

a) Write the differential equation of motion, and solve it for the time interval $0 < t < \pi\sqrt{m/k}$. (Note that the force term can be removed by shifting the origin of x by a certain amount.)

b) How large must A be if the mass comes to rest permanently at a distance B from the center, after crossing $x = 0$ 0, 1, 2, . . . times?

26-1. A man can walk at 5 ft/sec on a sidewalk, but only 3 ft/sec on a "uniformly rough" field. He starts at A, 140 ft from a wall, and goes to B, 120 ft south of the sidewalk along the wall.

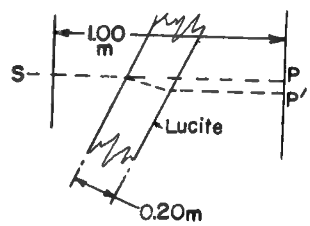



a) What route AKB must he follow to do this in least time?
(Note: It is legitimate to assume the "law of refraction" to apply to this problem. However, if you have enough courage, you might try to solve it without such an assumption!)

b) What is his least time?

c) What time do alternate routes ACB and AC'B require, if CK = KC' = 10 ft?

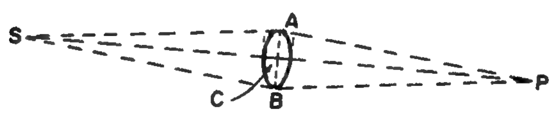
26-2. Light from source S sends a narrow beam perpendicular to a screen 1.0 m away. The ray strikes the screen at P. If a lucite slab of index of refraction 1.50 and thickness 0.20 m is inserted so that the beam strikes it at angle of incidence $30^{\circ}$,




a) find the lateral displacement of the ray, PP';

b) find the percent increase in time of path SP' over the original path in air, SP.

26-3.




S is a source of light. P is its image produced by a lens. SC = CP = 1.00 m. AC = BC = 0.10 m. The lens ACB is 3.0 mm thick at its edge. For the ray SCP to take just the same time as the rays SAP and SBP, how thick should the lens be at C? (Take index of refraction of glass as 1.60)

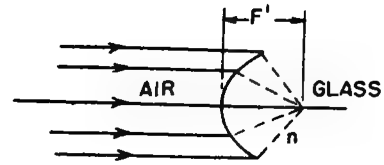
26-4. When we stand before an ordinary plane mirror, our image appears to be reversed, right for left: that is, the image of our right hand looks like the left hand of the "person" in the mirror. Why does the mirror not also reverse top for bottom? What <u>is</u> reversed by the mirror?

26-5. Two plane mirrors intersect each other so as to form a perfect internal right angle, with the line of intersection vertical. Explain why, in such a mirror, we "see ourselves as others see us."

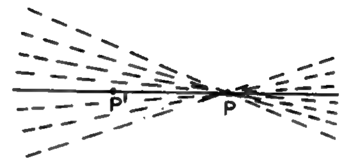
26-6. Three mutually perpendicular mirrors intersect so as to form an internal right-angled corner. A light ray strikes one of the mirrors, and thence perhaps one or both of the other two. Show that, after all reflections have occurred, (assuming the mirrors to be very large in extent) the ray is traveling exactly opposite to its original direction, but displaced laterally. Do you know of a practical application of such a "corner reflector"?

26-7. It is well known that when light goes from one transparent medium into another, not all of the light is refracted, but some is reflected, and very little, if any, is absorbed or scattered. What happens when a beam of light strikes the interface between two media, if the light beam is originally moving in the more dense medium at a large angle with the normal to the interface?

27-1. A parallel beam of light in air is to be brought to a point focus by a single refracting surface which bounds a region of index n. Find the proper shape for this surface.




27-2. A certain beam of light is converging toward a focus at a certain point P. It is desired to insert a single reflecting surface passing through a given axial point Q which will re-image the light to a new point focus at a given point P'. Find the shape of the required surface. Let the distance QP' = D and QP = d.




27-3. The outer diameter of a piece of glass capillary tubing is D, and its index of refraction is n. When viewed from the side, the small capillary bore appears to have a diameter d. What is its true diameter d'?

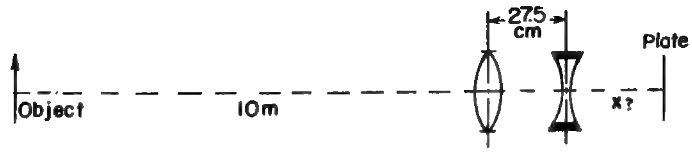
27-4. A lens of focal length F produces a real image of a distant object, and this image is viewed through a magnifying glass of focal length f. If the eye is focused at infinity while viewing, find the apparent angular magnification of the system.

27-5. The typical human eye can focus on objects lying between about 25 cm and infinity. A simple thin magnifying lens of focal length f = +5 cm is placed directly in front of the eye.

a) Between what two limiting positions should an object be placed to be seen clearly?

b) Determine the angular magnification for each of these positions.

27-6.




A telephoto combination consists of a positive lens of focal length $f_1 = +30$ cm and a negative lens of focal length $f_2 = -10$ cm. The separation between the two lenses is 27.5 cm. Where should a photographic plate be placed in order to photograph an object 10 meters in front of the first lens? Make a careful ray diagram.

27-7. The 200-inch Hale telescope, used in the Coude optical arrangement, has a focal length of 533 ft. What should be the distance between the focal plane for distant stars and

a) the moon;

b) an artificial earth satellite at a slant range of 200 mi?

27-8. Two thin lenses L and L', of focal lengths f and f', are separated by a distance D. Find the equivalent focal length F of the combination, and the distances $\Delta$, $\Delta'$ of the principal planes from the respective lenses L and L'.

28-1. Interpret the following two problems in complex numbers geometrically, and thus find the absolute value of A in each case:

a) $A = re^{i\theta/2} + re^{-i\theta/2}$

b) $A = \sum_{n=0}^{N} re^{in\theta}$

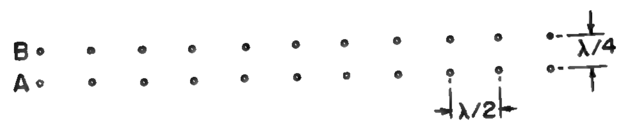
29-1. Two antennas are arranged as shown in Fig. 29-5(a) and are driven in phase. The antennas are driven so that one would, if alone, radiate a certain intensity $I_0$ in all horizontal directions, and the other, an intensity $2\ I_0$. What should the observed intensities in the various directions shown in the figure be?

29-2. Four identical dipole radiators are aligned parallel to one another and are equally spaced along a line at a distance 2.50 cm apart. They are driven at a frequency of $3.00 \times 10^9$ $\text{sec}^{-1}$ and are phased so that, starting from one end, each successive dipole lags the preceding one by $90^\circ$. Find the intensity pattern of the radiation at a great distance in the equatorial plane (perpendicular to the dipole axes), and sketch this function in polar coordinates. Such a diagram is called the radiation pattern of an antenna system.

29-3. The two paraboloidal "dishes" of the Caltech radio telescope in Owens Valley can be spaced at a distance of 1600 ft apart. Each dish concentrates the incoming radiation onto a small receiver at the focus of the paraboloid, and the two signals are fed into a single "mixer" located midway between the dishes. The mixer adds the two signals together and evaluates the mean-square amplitude of the resultant. How precisely should the angular position of a distant point source be determinable by this system, if a fluctuation of ten percent in the output signal is considered significant? Assume a wavelength of 50 cm.

29-4. A charge q traverses a circular path of radius a at an angular velocity $\omega$. Evaluate the electric field at a great distance R from the charge, at an angle $\Theta$ with respect to the axis of the circular path. Find the intensity of the radiation in the plane of the circle and on the axis, at a great distance R.

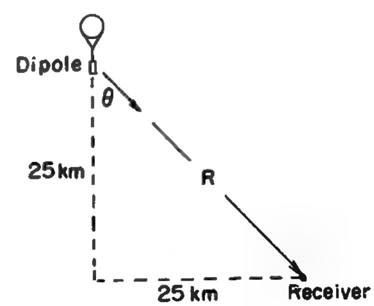
29-5.




A double line of N equally spaced oscillating dipoles is situated as shown. All dipoles in row A are driven in the same phase, and all those in row B lag $90^{\circ}$ in phase behind those of row A. Sketch the radiation pattern in the equatorial plane (as in Exercise 29-2) at a great distance from the array.

29-6. The electrons in a long, straight, fine wire of length L are all oscillating along the wire with angular frequency $\omega$, small amplitude a, and in the same phase. Find the electric field at a great distance $R (R \gg L)$, at an angle $\Theta$ with respect to an axis connected with the wire.

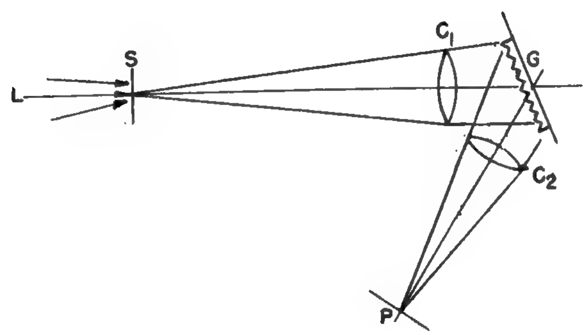
29-7. The power per unit area delivered by an electromagnetic wave is proportional to the mean-square electric field strength. Find what fraction of the total power radiated by an oscillating charge falls on a unit area normal to the radius vector R at an angle $\Theta$ with respect to the axis of oscillation. Evaluate this power in $Wm^{-2}$ for a vertically oriented dipole suspended from a cosmic ray radiosonde balloon at an altitude of 25 km and at a horizontal distance of 25 km from the receiver, if the transmitter is radiating 0.5 W total.



30-1. The wavelengths of the D-lines of sodium are 5889.95 Å and 5895.92 Å, respectively. How large a grating having 600 lines $mm^{-1}$ is needed to resolve these lines in the first order spectrum?

30-2. An automobile with the customary two headlights (considered as point sources) is approaching from a distance on a straight road. The lights on the car are situated 120 cm apart. How far from an observer would the car be when he could just be sure he was seeing two lights and not one? Take aperture of iris of eye as 0.5 cm and effective wavelength of light as 5500 AU. Would the fact that the light is "white" (a mixture of wavelengths) make it easier or harder to resolve the two sources?

30-3.




A common type of grating spectrograph is constructed as shown in the figure. Light from a source L passes through a narrow slit S, thence through a collimator lens (or mirror) $C_1$ which renders it parallel (so that it strikes the grating as would plane waves from infinity). This parallel light is then diffracted by the grating G; the diffracted light which proceeds in a certain range of angular directions strikes another lens $C_2$, called the camera lens, and is focused in a plane P, where the spectrum appears as a band, perhaps crossed by narrow spectrum lines at various places.

Let the length and width of the slit be h and w, the focal lengths of $C_1$ and $C_2$ be $F_1$ and $F_2$, the angles between the grating normal and the axes of $C_1$ and $C_2$ be $\Theta_i$ and $\Theta_d$, and the number of lines per mm on the grating be N.

a) How wide will the spectrum band appear at P?

b) What wavelength (s) will appear on the axis of $C_2$ at P?

c) How far apart in the focal plane at P will two spectral lines appear whose wavelength differs by 1.00 Å? This quantity is often called the <u>dispersion</u> of the instrument.

d) If the slit width w is much larger than the collimator lens resolution $1.22\ \lambda F_1/A_1$, where $A_1$ is the aperture, how wide should a spectral line at P be?

30-4.


The spectrograph at the 150 ft solar tower telescope of the Mt. Wilson Observatory is of the Littrow type, shown schematically in the figure. In this arrangement, a single lens acts as both the collimator and camera lens, and $\Theta_i = -\Theta_d$ (nearly). The spectrum is formed in a strip adjacent to the slit. The focal length of the Mt. Wilson instrument is F = 23 m, and the grating has a ruled area 15 cm × 25 cm with 600 lines $mm^{-1}$. The fifth order spectrum is commonly used.

a) At what angle $\Theta$ should the grating be tilted to bring the line $\lambda 5250.218$ of neutral iron in coincidence with the entrance slit in the fifth order spectrum?

b) What other wavelengths in the range $\lambda 3600$ - $\lambda 7000$ will also be coincident with the slit?

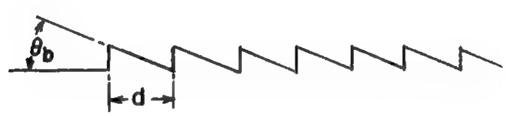
c) Suggest a simple way to remove the unwanted orders, leaving only the fifth order.

d) What is the dispersion of the instrument at fifth order $\lambda 5250$?

e) What is the minimum $\Delta\lambda$ which can theoretically be resolved at fifth order $\lambda 5250$ by this instrument?

30-5. The wavelengths of spectral lines are commonly measured to 0.001 Å using spectrographs whose resolving power may be only .010 Å. Are any basic laws of physics being violated in the process? Explain.

30-6.

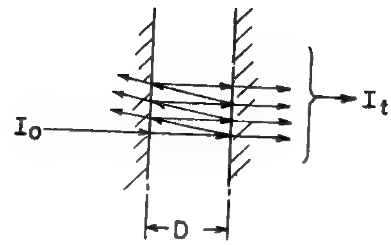



When the grooves of a diffraction grating are shaped in such a way as to throw most of the incident radiation into a particular direction, the grating is said to be <u>blazed</u> for this direction. Suppose it were possible to shape the grooves perfectly in a saw-tooth shape as shown, each groove surface being tilted at a certain angle $\Theta_b$.

a) Use the notion of the diffracted beam being the radiation emitted by oscillators in the material, which are driven in phase with the incoming radiation, to deduce in what direction the diffracted beam would be most intense if $\Theta_i = 0$.

b) Estimate the approximate angular range over which the blaze would extend.

30-7.




A Fabry-Perot interferometer consists of a pair of very accurately flat surfaces, parallel to each other at a distance D apart. The surfaces are coated so as to reflect a fraction $R^2$ of light incident on them normally, and to transmit a fraction $T^2$. Light of intensity $I_o$ and wavelength $\lambda$ is incident upon one surface from the left (see figure). Part of this beam is transmitted directly through the system, but some of the light is reflected from the second surface, then from the first surface, and is thence transmitted. In general, the outgoing beam is made up of light which has been reflected 0, 2, 4, 6, . . . times and transmitted through 2 films, all summed together. How should the transmitted intensity depend upon D, $\lambda$, R, and T?

Note: Narrow-band optical filters, called interference filters, operate on this same principle, but the two reflecting surfaces are made by high-vacuum coating a piece of glass with several layers, accurately controlled in thickness, of clear materials having various indexes of refraction.

31-1. Find the index of refraction of Aluminum for x-rays of wavelength $1.56 \times 10^{-8}$ cm. Assume that all of the electrons in Aluminum have natural frequencies very much less than the frequency of the x-rays.

31-2. The index of refraction of the ionosphere for radio waves of frequency 100 Mc sec$^{-1}$ is $n = 0.90$. Find the density of electrons per cubic centimeter in the ionosphere.

31-3. Noting that the electric field of a light wave traveling through a medium of refractive index n is $E = E_o e^{i\omega(t - nz/c)}$,

a) Show that, if $n = n' - in''$,

$$E = E_o e^{-n''\omega z/c} e^{i\omega(t - n'z/c)}$$

b) Use the equation $n - 1 = \frac{Nq^2}{2\epsilon_o m} \frac{1}{\omega_o^2 - \omega^2 + i\gamma\omega}$

to find the rate at which the intensity of a beam of radiation whose frequency is exactly equal to the natural frequency $\omega_o$ of an atom is attenuated.

31-4. It was deduced that the instantaneous energy flux of a wave is $S = \epsilon_o c E^2$ watts per square meter:

a) Find the total rate at which energy is radiated by an electron which is oscillating with amplitude $x_o$ and angular frequency $\omega$.

b) Compare the energy radiated per cycle with the stored energy $\frac{1}{2}m\omega^2 x_o^2$, and thus find the damping constant $\gamma_R$. This process is called radiation damping.

c) An excited atom gives out radiation having a certain wavelength $\lambda$. Calculate theoretically the expected breadth $\Delta\lambda$ of the spectral line, if the breadth arises solely from radiation damping. (Think of the atom as being a tiny damped oscillator having a large Q.)

32-1. Show that if the equation of motion of a charged oscillator is assumed to be

$$md^2x/dt^2 + \omega_o^2 x - (2e^2/3c^3)d^3x/dt^3 = F(t)$$

the third-derivative term will correctly describe the rate of loss of energy by radiation (the radiation resistance) at any frequency.

Hint: Assume $F(t) = A \cos \omega t$ and find the amount of power absorbed from the driving source.

32-2. A beam of light is passing through a region containing N scatterers per unit volume, each of which scatters the light with a cross section $\sigma$. Show that the intensity of light as a function of the distance, x, traversed is

$$I = I_o e^{-N\sigma x}$$

32-3. Using the scattering formula

$$\sigma = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 \frac{\omega^4}{(\omega^2 - \omega_o^2)^2}$$

and the formula previously derived for the index of refraction of a gas, show that the quantity $N\sigma$ can be written as

$$N\sigma = \frac{2}{3\pi}\,\frac{(n-1)^2}{N}\left(\frac{2\pi}{\lambda}\right)^4$$

(This was one of the first ways used to estimate Avogardro's number, using the scattering of the blue sky.)

32-4. How much is the blue light ($\lambda$ = 4500 Å) of the sun attenuated in going through the atmosphere when the sun is

a) at the zenith, and

b) $10^o$ from the horizon?

32-5. A new radiation is discovered (called x-rays because they are new and mysterious), and are suspected of being transverse waves like light. Scattering of these rays by electrons in matter has been observed. How could you prove they are transverse waves and can be polarized?

32-6. The inner corona of the sun (called the K-corona) is sunlight that has been scattered by free electrons. The apparent brightness of the K-corona at one solar radius from the sun's limb is about $10^{-8}$ that of the solar disc, (per unit area). Estimate the number of free electrons per $cm^3$ near the sun.

32-7. Show that the quantity $(\epsilon_0 c)^{-1}$ has the dimensions of resistance, and evaluate it numerically.

32-8. Interstellar space is populated by clouds of tiny dust grains composed of carbon, ice, and small amounts of other elements. Estimate the minimum mass per unit area ($g\ cm^{-2}$) of such dust needed to obscure our view of stars behind it by, say a factor 100 (i.e., 5.0 stellar magnitudes). Note that the dust grains may remove starlight by scattering as well as by simple absorption.

32-9. A short straight piece of copper wire placed in the beam of electromagnetic waves sent out by a radar antenna will "scatter" some of the wave. The electric field of the wave sets up a motion of the electrons in the wire and this motion results in the radiation of the "scattered" wave. For a short piece of wire (length $\ll \lambda$) we can assume that the average displacement of the electrons in the wire is along the axis of the wire and is proportional to the component of the electric field parallel to the wire. That is, if there are N electrons in the wire and d is their average instantaneous displacement, then $d = XE_{\parallel}$, where $E_{\parallel}$ is the component

of the electric field of the wave parallel to the wire. We would like to know (in terms of $\lambda$ and $N$):

a) What is the maximum scattering cross-section of the wire?

b) How does the scattering cross-section depend on the orientation of the wire?

33-1. Two polaroid sheets are set with their axes at right angles. A third sheet is then inserted between them with its axis at an angle $\theta$ with the first polaroid. What intensity should be transmitted by this combination, if the polaroids are ideal (lossless)?

33-2. Assume that when a beam of plane polarized light strikes a polaroid sheet, a fraction $\alpha^2$ of the intensity is transmitted if the polaroid axis is parallel to the polarization axis, and a fraction $\epsilon^2$ is transmitted if the two axes are at right angles. (If the polaroid were ideal, $\alpha^2$ would be unity and $\epsilon^2$ would be zero). Unpolarized light of intensity $I_o$ is normally incident on a pair of polaroid sheets with an angle $\theta$ between their axes. What intensity should be transmitted? (Ignore reflection effects)

33-3. Show that Brewster's angle (the angle of incidence <u>i</u> for which the reflected ray is linearly polarized) is such that $\tan i = n$.

33-4. Discuss the intensity and polarization of the radiation emitted by an electron moving at constant speed in a circular path, particularly for points (a) on the axis of the circle; and (b) in the plane of the circle.

33-5. The indices of refraction of crystalline quartz for light of wavelength 600 mμ are $n_o = 1.544$ and $n_e = 1.553$, for the ordinary and extraordinary rays respectively. If a crystal of quartz is cut parallel to its optic axis, one may take advantage of the maximum difference in speed of the ordinary and extraordinary rays as they enter normally and progress through the crystal. What thickness of crystal is required to shift the relative phases of these two rays by $90^o$, for light of the above wavelength?

33-6. A vacationing freshman from CIT, strolling with a girl friend, sees the moon, $10^{\circ}$ above the horizon, reflected in a calm lake. Nostalgically recalling chapter 33, he attempts to calculate how bright the image should appear, compared with the moon itself. He assumes the radiation from the moon to be (nearly) unpolarized. What result should he expect to get?

Show that the reflected intensity approaches 100 per cent for grazing incidence.

33-7. If light falls perpendicularly on the plane facet of a diamond ($n = 2.40$),

a) what fraction of the incident radiation is reflected?

b) what is Brewster's angle for diamond?

33-8. In extension of Problem 5, suppose the indices of refraction for light of wavelength 410 millimicrons are $n_o = 1.557$ and $n_e = 1.567$, and that the crystal of quartz is cut as a quarter-wave-plate for wavelength 600 millimicrons. Describe fully the state of polarization of emergent light of this shorter wavelength which is linearly polarized before entry into the crystal.

33-9. You are given a polished plate of black obsidian and are asked to measure the index of refraction of the material. How would you proceed, and what precision would you expect to attain?

34-1. A disc of radius A rolls without slipping on a horizontal plane. Write the equations of the path followed by a point at a radius $R \leq A$ from the center of the disc in terms of A, R, and the angle $\theta$ through which the disc has turned. Let x be measured from the center of the disc vertically and z be measured horizontally.

34-2. If $Z = ct$, find the transverse acceleration of $d^2x/dt^2$ of the point. This is the retarded acceleration needed for calculating the radiation from a particle moving in a circular path of radius R. Express the result in terms of the observable quantities R, v (the speed of the particle in its path), and x (the apparent transverse position of the particle at the time of observation).

34-3. Find the ratio of the radiation intensities that will be observed as the particle moves toward and away from the observer in its circular path.

34-4. As suggested on p. 34-10, derive the expression $\sin\theta = v/c$ using the Lorentz transformation.

34-5. Show that the speed of a 1 GeV electron differs from c by one part in $8 \times 10^6$.

34-6. The D-lines of sodium (laboratory wavelength, 589.0 m $\mu$) are observed to be shifted to 588.0 m $\mu$ in the spectrum from a certain star. What is the star's velocity relative to the observer? Is the relativity correction needed?

34-7. The Caltech astronomer R. Minkowski concluded that the most distant nebula on which he made observations was receding at a speed of 0.6 c. What Doppler shift would be observed in the light from such a nebula? Find the observed wavelength of a spectrum line whose laboratory wavelength is measured as 300 m $\mu$.

34-8. Bradley (1728) observed the aberration of light by which stars appear to be displaced in the sky because of the earth's motion in its orbit. The telescope must be "directed forward" a maximum of 20.5" arc for stars near the pole of the ecliptic. If one considers the velocity of light as known, $3.00 \times 10^{8}$ m/sec, to what value of the radius of earth's orbit does this observation lead?

34-9. Assume that interplanetary space is populated by small grains of "dust," of mean specific gravity $\rho$ and of roughly spherical shape of radius R.

a) Show that, for any size dust grain, the ratio of the gravitational attraction toward the sun to the radiation pressure away from the sun is independent of the distance from the sun.

b) Using the fact that the solar radiation intensity at the earth's orbit is 1374 W $m^{-2}$, and assuming the absorption cross-section to be $\pi R^2$, find for what radius R the radiation pressure and gravitational attraction will just balance.

c) Considering the results of Chapter 32, can the effective cross section of a dust grain be appreciably greater than $\pi R^2$?

34-10. In one proposed means of space propulsion, a thin sheet of highly reflecting plastic film will be used as a radiation pressure "sail." A plane sheet 100 m square is available, and the mass of the space ship is $10^3$ kg. If the space ship initially travels in a circular orbit of 1 A.U. radius about the sun, describe how to use the "sail" to increase the mean radius of the orbit, and find at what rate the orbit radius will grow.

38-1. An orchardist found it was easy to set two trees in line, but harder to set three. However, by practice and careful surveying, he set out 64 small trees on a square E-W, N-S grid, 8 trees to a row, and 8 rows, with a $6.0 \times 6.0$ m basic square. Standing at one corner of his orchard, he observed 3 lines of 8 trees each, counting the tree in the corner where he stood, 2 lines of 4 each, and 4 lines of 3 each.

a) What was the least angle between two adjacent lines of these nine lines?

b) What was the greatest distance between two successive trees in any one of these lines?

c) In an "infinite orchard" set out on this basic grid, each of these lines would appear from the air as one of a set of parallel lines well populated with trees. The distance between adjacent parallel lines of any set could be considered as its "grating space." Find the grating space for successive sets from the south front of the orchard to the $45^\circ$ line.

38-2. The sodium and chlorine atoms of an NaCl crystal are alternately spaced at the corners of a cubical lattice whose Na-to-Cl closest spacing is $d = 2.82$ Å. Find the largest five interplanar spacings for the NaCl crystal, and find at what angles first-order Bragg reflections should occur for these planes, if X-rays of wavelength 1.50 Å were used.

38-3. In Chapter 32 (p. 32-3) we learned that an excited atom would radiate away its energy at a certain rate, which has the effect both of limiting the "life time" of an excited state and of

introducing a finite width to the corresponding spectral line. Show that these effects, interpreted as uncertainties in the energy and the time of measurement of a photon (or of momentum and position) are consistent with the uncertainty principle.

38-4. a) Check by your own dimensional analysis the "Bohr radius" of the hydrogen atom.

b) Show by the Uncertainty Principle that the energy needed to remove an electron from its associated proton in hydrogen is on the order of a few electron volts.

38-5. In the ultraviolet spectrum of hydrogen there is observed a series of lines known as the Lyman series. The wavelengths in Ångstrom units for the three longest lines of this series are as follows:

1216, 1026, 973

Compute the wavelengths of three other possible lines in the spectrum of hydrogen that could be "predicted" on the basis of this information alone, together with the Ritz combination principle. Two of these lines are in the visible region (Balmer series), and one is a line in the infrared (the first line of the Paschen series).

39-1. If an ideal gas is compressed adiabatically, we have found (Eq. 39-14) that $PV^\gamma$ = constant. On the other hand, under all conditions, PV/T = constant. Combine these to deduce how P and T, or V and T, are connected during an adiabatic compression.

39-2. A bicycle pump is being used to inflate a tire to a pressure of 50 lb $in^{-2}$ gauge pressure, starting with air at atmospheric pressure, 14.7 lb $in^{-2}$ at $20^\circ C$ ($293^\circ K$). If $\gamma = 1.40$ for air, at what temperature centigrade is the air as it leaves the pump? Neglect heat losses to the walls of the pump.

39-3.


Helium gas is contained in one half of each of two identical containers, and the other half of each container is evacuated. The two halves of each container are separated by a piston which has a small stopcock through it. (See the figure above)
Two experiments are now performed:

a) The stopcock is opened in one piston and the gas is allowed to flow through to the other side until equilibrium is reached. The piston is then very slowly moved to one end of the container.

b) The piston of the other container is very slowly allowed to move into the evacuated end of the container, and then the stopcock is opened. Compare quantitatively the final state of the gas in the two containers. (Ignore heat loss through walls, and friction).

39-4. a) Imagine a tall vertical column of gaseous or liquid fluid whose density varies with height. Show that the pressure as a function of height follows the differential equation $dP/dh = -\rho(h)g$.

b) Solve this differential equation for the case of a gaseous atmosphere of molecular weight $\mu$, in which the temperature is constant as a function of h.

39-5. An atmosphere in which the pressure and density as a function of height satisfy the relation $P\rho^{-\gamma}$ = constant is called an adiabatic atmosphere.

a) Show that the temperature of such an atmosphere decreases linearly with height, and find the constant of proportionality. This temperature gradient is called the <u>adiabatic lapse rate</u>. Evaluate this temperature gradient for the earth's atmosphere.

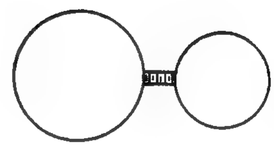
b) Use an argument based on energy considerations to show that an atmosphere having less or more temperature gradient than the adiabatic lapse rate will be stable or unstable against convection, respectively.

39-6. A cylinder with a leakless, frictionless piston contains 1 $m^3$ of a monatomic gas ($\gamma = 5/3$) at gauge pressure 1 atmosphere ($1.01 \times 10^5$ N $m^{-2}$). The gas is slowly compressed at constant temperature until the final volume is only 0.4 $m^3$. How much work must be done to accomplish this compression?

39-7. Two samples of gas, A and B, of the same initial volume, $V_o$, and at the same initial absolute pressure, $P_o$, are suddenly compressed adiabatically, each to one-half its initial volume. How does the final pressure of each sample compare with its initial pressure, if $\gamma_A$ is 5/3 (monatomic) and $\gamma_B$ is 7/5 (diatomic)?

39-8. Find the ratio of works required to perform the two compressions described in Problem 7.

39-9.* Two bulbs of volumes 200 $cm^3$ and 100 $cm^3$ respectively are connected by a short tube containing an insulating porous plug that permits equalization of pressure but not of temperature between the bulbs. The system is sealed at $27^\circ$ C when it contains oxygen under a pressure of 760 mm Hg. The small bulb is immersed in an ice bath at $0^\circ$ C and the large bulb is placed in a steam bath at $100^\circ$ C. What is the final pressure inside the system? Neglect thermal expansion of the bulbs.



39-10.* A 50 liter tank is connected to a 15 liter tank through a short tube which contains a pressure release valve which will only allow gas to pass from the larger tank to the smaller tank if the pressure in the larger tank exceeds the pressure in the smaller tank by 88 cm of Hg. If at $17^\circ$ C the larger tank contains gas at atmospheric pressure and the smaller tank is evacuated, what will be the pressure in the smaller tank when both tanks are $162^\circ$ C?

39-11.* At ordinary temperature nitrogen tetroxide is partially dissociated into nitrogen dioxide as follows:

$$N_2O_4 \rightleftarrows 2\ NO_2$$

Into an evacuated flask of 250 $cm^3$ volume 0.90 g of liquid $N_2O_4$ at $0^\circ C$ is introduced. When the temperature in the bulb has risen to $27^\circ C$ the liquid has all vaporized and the pressure is 960 mm Hg. What percent of the nitrogen tetroxide has dissociated?

39-12.* A mole of ideal monatomic gas in a non-insulated container with a movable piston is originally at $P_1$, $V_1$, and $T_1 = 27^\circ C$. Then the gas is slowly heated using a total of 8.31 W hr of energy and at the same time allowed to expand at constant pressure to a new state $P_1$, $V_2$, and $T_2$. From evaluation of the new energy content of the gas and the work done by the gas during expansion, find

a) $T_2$

b) $V_2/V_1$

# CHAPTER 40

40-1. In a radiometer, the molecules of a gas at low pressure bombard a set of thin, light vanes which are black on one side and shiny on the other. When radiation strikes these vanes, the absorbed energy is carried away principally by the molecules striking the blackened side of each vane, and the vanes turn as a result of this unbalanced force. Consider a vessel in which there are n molecules of mass m per unit volume, at an absolute temperature T. A thin vane of unit area inside the vessel is absorbing radiant energy at a rate $\Pi$ watt, and this energy is being carried off (isotropically) by the molecules striking one side of the vane. Estimate roughly the unbalanced force on the vane for air at room temperature.

40-2. In a gas at thermal equilibrium, what fraction of the molecules striking a surface have kinetic energies greater than a) average? b) three times the average?

40-3. The molar heat capacity at constant volume of a substance, $C_V$, is the amount of energy needed to raise the temperature of one mole of the substance by one degree, while holding the volume constant. What is the molar heat capacity at a constant volume of a) an ideal monatomic gas? b) an ideal diatomic gas?

40-4. Air at NTP is flowing at speed v through a smooth pipe of constant cross-sectional area A. As it passes a wire grid which offers

negligible resistance to the flow, it is heated; the energy input is P watts. It ultimately emerges from the tube at a speed $v'$. Write equations for conservation of mass, energy, and momentum as the air traverses the tube, and thus find:

a) $v'$

b) the final temperature T

c) the thrust* F.

* This is basically a jet engine.

40-5. Discuss the approximate performance of an aircraft jet engine in the light of the above problem, if the engine consumes 100 kg of air and 2.00 kg of kerosene per second. The heat of combustion of kerosene is about $4.65 \times 10^7$ J kg$^{-1}$. What complications might invalidate your result?

40-6. The Maxwellian distribution law is of the general form $dN/dv = Av^2e^{-bv^2}$. This may be transformed to $y = x^2e^{-x^2}$.

a) Graph this equation for $0 \le x \le 3.0$ to show how an increasing $y = x^2$ curve is suppressed by the exponential.

b) Find its maximum ordinate.

c) See how closely the area under your curve comes to

$$\int_0^\infty x^2e^{-x^2}\,dx$$

40-7. In the atmospheric pressure law $n = n_o e^{-\frac{mgh}{kT}}$; $\frac{kT}{mg} = \frac{RT}{ug} = h_o$ is called the <u>scale height</u>, where u is the molecular weight. Evaluate the scale height for the earth's atmosphere and the sun's atmosphere, given $u_\oplus = 29$, $T_\oplus = 300^\circ$K, $u_\odot = 1.5$, $T_\odot = 5500^\circ$K, $g_\odot = 2.7 \times 10^2$ m s$^{-2}$.

# CHAPTER 41

41-1. Calculate (and remember) the following:

a) The temperature T for which kT is equal to 1 electron volt.

b) The value of kT in electron volts for room temperature.

c) The wavelength of a photon corresponding to a quantum jump of 1 eV.

41-2. The distribution law for black-body radiation is:

$$I(\omega)d\omega = \frac{\hbar\,\omega^3 d\omega}{\pi^2 c^2 (e^{\hbar\omega/kT}-1)}$$

By changing the variable from $\omega$ to $x = \hbar\omega/kT$, show that:

a) The total radiation intensity, integrated over all frequencies, is proportional to the fourth power of the absolute temperature.

b) The frequency $\omega_m$ at which $I(\omega)$ has its maximum value is proportional to the absolute temperature.

41-3. Find the relative intensities of light of wavelength 0.31 $\mu$ for two black-body sources at temperatures of 2000°K and 4000°K.

# CHAPTER 42

42-1. Activation energies, heats of vaporization, heats of formation or dissociation, etc., are commonly expressed in Joules per g-mole or in electron volts per atom. How many Joules per g-mole is 1 eV/atom?

(Note: Chemists use an energy unit called a kilocalorie. 1 kilocalorie = 4186 J.)

42-2. a) Plot the vapor density of mercury vs 1/T on semi-log paper, (data are available in the Handbook of Chemistry and Physics) and from this plot, deduce the heat of vaporization of mercury. Check your result against the tabulated value.

b) Do the same for water.

42-3. Over the temperature range 0 - 300°C the heat of vaporization of mercury changes by only 3 per cent, and is about 0.61 eV/atom on the average. How much error will one make in calculating the vapor density of mercury at 0°C, if the heat of vaporization at 300°C is used instead of the correct 0°C value?

Moral: A small percentage error in a large exponent can have a large effect.

42-4. The resistivity of nearly pure silicon as a function of temperature

is shown in Fig. 42-4. Make a quantitative deduction concerning the nature of the current flow in this substance above and below 300°C.

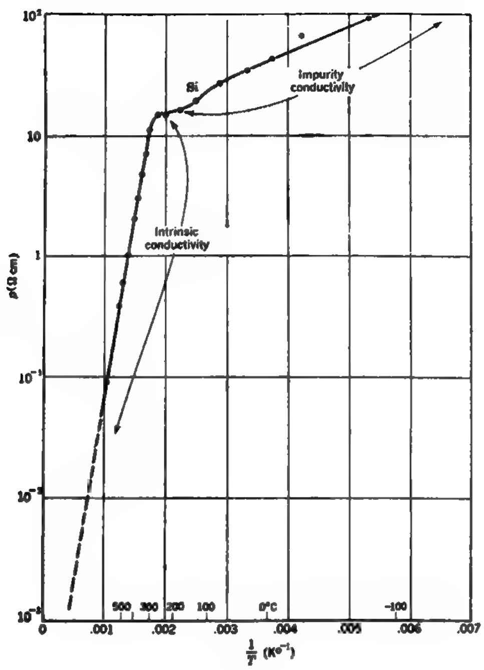



Fig. 42-4

43-1. The "diameter" of an oxygen molecule is roughly 3Å. Estimate the mean free path and the mean time between collisions for oxygen gas at NTP.

43-2. A certain vessel contains $10^{24}$ molecules of a gas for which the mean free path is $\ell$. For what path length L is there less than a 50 per cent chance that any of the molecules in the container will go farther than L before it suffers its next collision?

43-3. When a temperature gradient exists in a material, an energy flow proportional to the temperature gradient results. (Ignore convection). The coefficient of proportionality, reduced to a unit area and unit temperature gradient, is called the thermal conductivity, K. Thus $dE/dt = KA\,dT/dx$. Show that, in the absence of convection, the thermal conductivity of a gas is

$$K = kn_0 v\ell/(\gamma - 1) = kv/(\gamma - 1)\,\sigma$$

Hint: Interpret thermal conductivity as a transport of internal (heat) energy U across a plane, from one mean free path on either side, as was done for diffusion.

43-4. When a velocity gradient exists in a fluid, such that the velocity changes with distance at right angles to the flow direction, a drag results, called viscosity. In a gas, this is due to the transport of momentum across a plane, from roughly one mean free path on either side. If the flow is in the x-direction and there is a gradient of $v_x$ in the y-direction, then the drag force per

unit area on a plane perpendicular to y is

$$F/A = \eta \, dv_x/dy$$

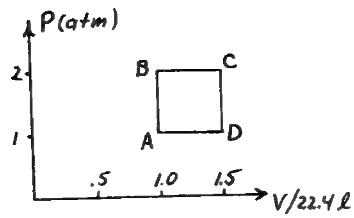
Show that, for a gas the coefficient of viscosity, $\eta$, is approximately,

$$\eta = n_o vm\ell$$
$$= vm/\sigma$$

43-5. Notice that the thermal conductivity and the viscosity of a gas are both independent of the pressure. Devise a suitable modification of the formulas for the energy transfer between two surfaces whose temperature are T and T + $\Delta T$, situated a distance D apart, if $\ell \gg D$. Do the same for the momentum transfer between two such surfaces, moving at speeds U and U + $\Delta U$.

43-6. Two gases, A and B, are at density $\rho_A$ and $\rho_B$ at a certain temperature $T_o$. A particular ion is observed to have a mobility $\mu_a$ in gas A, and $\mu_b$ in gas B. What mobility would you expect the ion to have in a mixture of these gases, at density $\rho_A + \rho_B$ and temperature $T_o$?

# CHAPTER 44

44-1. 


A sample of ideal gas with $\gamma = 4/3$ is taken successively from condition A (1 atm press. 22.4 liter vol. at $300^\circ$K) to condition C (2 atm press., 33.6 liter vol. at $900^\circ$K) by two routes, ABC and ADC.

a) Show that the change in entropy is the same by both paths.

b) Compute this change.

44-2. Translate the ideal Carnot cycle abcd on a p-V diagram between $T_1$ and $T_2$ and $(p_a, V_a)$ $(p_c, V_c)$ into a temperature entropy diagram with corresponding points ABCD. (See Fig. 44-6)

44-3.✓ In a modern steam power plant using superheated steam, the temperature in the steam generator is $600^\circ$C. The intake river water used to cool the condenser is at $20^\circ$C. What maximum efficiency could such a plant have?

44-4. In an ideal reversible engine employing 28 g nitrogen as working substance ($\gamma = 7/5$) in a cyclic operation abcd without valves as in Fig. 44-6. The temperature source is $400^\circ$K, of sink $300^\circ$K. The initial volume of gas at point A was 6.0 liters and the volume at point C is 18.0 liters.

a) At what volume $V_b$ should the cylinder be changed from heat input (isothermal expansion) to isolation and adiabatic expansion (from $V_b$ to $V_c$)? At what $V_d$ should the adiabatic compression begin?

b) How much heat is put in on the $V_a \rightarrow V_b$ part of the cycle?

c) How much heat is extracted during the $V_c \rightarrow V_d$ part?

d) What is the efficiency of the engine?

e) What change in entropy per gram occurs in the working substance during $a \rightarrow b$? $c \rightarrow d$?

Hint: You should find that in a Carnot cycle for an ideal gas the expansion ratios $V_b/V_a$ and $V_c/V_d$ are equal.

44-5. A careless experimenter left the valve of a tank of helium slightly open over the weekend. The gas, originally at 200 atm slowly escaped isothermally at $20^{\circ}$C. What change in entropy per kg of gas occurred?

45-1. The sun radiates approximately like a black body of temperature $5700^{\circ}K$. If a perfectly black copper sphere is irradiated by sunlight at a distance of one astronomical unit from the sun, what equilibrium temperature will it attain? (The sun's diameter subtends an angle of $0^{\circ}.50$ at the earth.)

45-2. Sunlight beats down perpendicularly on a large black paved field in Equatorial Africa. If the surface radiates as a black body, what maximum temperature does it attain? (Solar constant, 1395 watts/$m^2$)

45-3. A black body of radius r at temperature T is surrounded by a thin shell of radius R, black on both sides. Find by what factor this radiation shield reduces the rate of cooling of the body.

(Consider space between sphere evacuated, with no thermal conduction losses).

45-4. The density at the center of the sun is about 80 g $cm^{-3}$ and the central temperature is about $13 \times 10^{6}$ $^{\circ}K$. The matter is composed almost entirely of protons and electrons. Find the gas pressure and the radiation pressure at the center of the sun.

45-5. The latent heat of vaporization of water is about $2.44 \times 10^{6}$ J $kg^{-1}$, and the vapor density at $100^{\circ}C$ is 0.598 kg $m^{-3}$. Use the Clausius-Clapeyron Equation to find the rate of change of the

boiling temperature with altitude near sea level in $^{\circ}C$ per km. Assume the temperature of the air is $300^{\circ}K$.

45-6. For an ideal gas, whose internal energy depends only upon T, show that the difference between the molar heat capacities at constant pressure and at constant volume is equal to the gas constant R: $C_p - C_v = R$.

45-7. At $0^{\circ}C$, the specific volume of saturated water vapor is 206 $m^3/kg$. What is the latent heat of vaporization in $J/kg^{-1}$ at this temperature? (Determine dp/dT from tables, calculate L, compare with tabular value.)

45-8. If a certain object absorbs a fixed fraction A of all radiation incident upon its surface and reflects the rest, show that, at temperature T, it emits an amount $A\ \sigma T^4$ per unit area.

45-9. a) Use a thermodynamic argument to show that, if a substance expands when it freezes, its freezing temperature must decrease with increasing pressure.

b) Estimate the lowest temperature of the ice on a skating rink for which ice skating would be possible.

## CHAPTER 47

47-1. Find the ratio of the speed of sound in helium to that in hydrogen at the same temperature.

47-2. Two whistles of the same length are blown (a) with air cooled to liquid air temperature ($-180^{\circ}C$) and (b) with heated air. One whistle gives a pitch just one octave above the other (twice the frequency). What should be the temperature of the air blowing the whistle (b)?

47-3. If one inhales helium and speaks, his voice sounds unnatural and high in pitch. If all your resonant cavities ("the empty parts of your head") were filled with helium instead of air, by about what factor would every <u>resonant</u> frequency be increased? If you were to sing a tune, what effect would the helium have on the key in which you sing? Discuss.

47-4. Consider a steady, plane sound wave of frequency 1000 $sec^{-1}$ in which the pressure peaks are $\pm 1$ dyne $cm^{-2}$ from the prevailing atmospheric pressure of $1 \times 10^{6}$ dynes $cm^{-2}$.

a) What change in density accompanies such a wave?

b) What maximum particle displacement $X_m$?

c) What intensity?

(Take velocity of sound as 340 m/sec.)

47-5. Pinch a single length of rubber band about 5 cm long between the fingernails of your two hands; twang it to observe the pitch; then stretch it 2×, 3×, 4×, 5× its original length <u>without changing the mass of band between fingernails</u>, twanging it

as you proceed. Discuss the results observed. Why does not a violin string do the same thing?

47-6. A uniform, perfectly flexible string of density $\sigma$ kg $m^{-1}$ is stretched with a tension T. Derive the wave equation governing the lateral displacement y and deduce the speed of propagation of disturbances along the string. Assume that $\partial y/\partial x \ll 1$ at all points and times, and consider only vibration in a plane. Note that the component of the string tension in the lateral direction is very nearly $T\,\partial y/dx$.

47-7. Show that $u = Ae^{i(\omega t - kx)}$ satisfies the wave equation,

$$\partial^2 u/\partial x^2 = (1/v^2)\,\partial^2 u/\partial t^2$$

provided $\omega$ and k satisfy the relation $\omega = vk$.

# CHAPTER 48

48-1. The phase velocity of a water wave of wavelength $\lambda$ is, neglecting surface tension and the effects of finite depth,

$$V_{ph} = \sqrt{g\lambda/2\pi}$$

Show that the group velocity is one half the phase velocity. What are the group and phase velocities of a wave of wavelength 1000 m?

48-2. If surface tension is included, the phase velocity of a surface wave on a liquid of density $\rho$ and surface tension T is

$$V_{ph} = \left(\frac{2\pi T}{\lambda\rho} + \frac{g\lambda}{2\pi}\right)^{\frac{1}{2}}$$

if the depth is sufficiently great. Find the group velocity of such a wave.

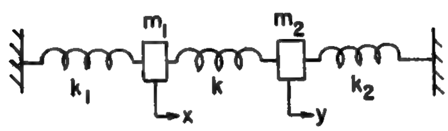
48-3. Find the phase velocity of ripples of wavelength 1.0 cm a) in water (surface tension 70 dyne $cm^{-1}$); b) in alcohol (surface tension 26 dyne $cm^{-1}$).

48-4. Find the wavelength and frequency of ripples on water which advance with minimum speed.

48-5. A long diesel freight train is travelling uphill at a speed of 5.0 m $sec^{-1}$ on a straight track. As it approaches a tunnel in a sheer vertical wall, the engineer gives a long steady blast of the horn, whose principal frequency is 340 cycles $sec^{-1}$. Both the horn itself and its echo from the wall are heard a) by the engineer; b) by a person on the ground near the caboose. How many beats per second does each person hear?

49-1. Two frictionless gliders of mass $m_1$ and $m_2$ are attached to two opposite walls by springs of stiffness $k_1$ and $k_2$, respectively, and are also attached to one another by a spring of stiffness k. (See the figure.) Write down the equations of motion of the two gliders. Let

$$k_1/m_1 = k_2/m_2 = \omega_o^2$$




49-2. Substitute $x = Ae^{i\omega t}$ and $y = Be^{i\omega t}$ into the above equations and find the frequencies and amplitudes of the two normal modes of vibration.

49-3. Show that the function

$$f(x,y,z,t) = Ae^{i\omega t} \sin l\pi x/a \sin m\pi y/b \sin n\pi z/c$$

where

$$\omega^2 = v^2\pi^2(l^2/a^2 + m^2/b^2 + n^2/c^2)$$

and l, m, n are integers $\geq 1$,

a) satisfies the three-dimensional wave equation (with propagation velocity v).

b) is equal to zero at $x = 0$ and $x = a$, $y = 0$ and $y = b$, and $z = 0$ and $z = c$.

c) oscillates sinusoidally in time.

49-4. If $a:b:c = 1:2:3$ in the previous problem, evaluate the lowest ten frequencies in terms of the lowest frequency $\omega_o$. List them in order of increasing frequency and plot them roughly on a vertical scale.

49-5. Use the idea of infinitely long, periodic wave trains moving in opposite directions to deduce what will happen if an ideal, uniform stretched string of length L is pulled aside a distance A at its midpoint and then released. Sketch a few representative views of the appearance of the string at various times throughout a half-cycle of the motion.

# CHAPTER 50

50-1. Starting from the Fourier analysis of the square wave,

$f(x) =$ 


$$= \frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \ldots\right)$$

Show that

a) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots = \pi/4$

b) $1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \ldots = \pi^2/8$

c) $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \ldots = 4/3 \times \left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \ldots\right) = \pi^2/6$

50-2. Analyze the function

$g(x) =$ 


into its Fourier components and show that your result agrees with the result obtained by integrating the function of the previous problem.

50-3. Use the result of the previous problem to show that

a) $1 + 1/3^4 + 1/5^4 + 1/7^4 + \ldots = \pi^4/96$

b) $\sum_{n=1}^{\infty} (1/n^4) = \frac{2^4}{2^4 - 1}\left(1 + 1/3^4 + 1/5^4 + \ldots\right) = \pi^4/90$

50-4. In Chapter 45 we needed to evaluate $\int_0^\infty \frac{x^3\,dx}{e^x - 1}$ . You can now do this by multiplying numerator and denominator by $e^{-x}$ and expanding

$$\frac{1}{1 - e^{-x}} = (1 + e^{-x} + e^{-2x} + \ldots)$$

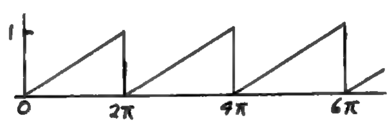
and carrying out the integrals term by term. Thus you should obtain

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \int_0^\infty u^3 e^{-u}\,du\,[1 + 1/2^4 + 1/3^4 + \ldots]$$

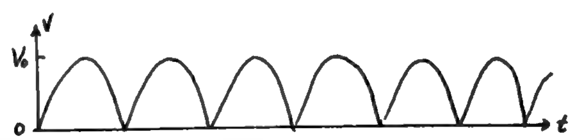
$$= 6 \times \pi^4/90 = \pi^4/15.$$

Try it.

50-5. Find the Fourier series which represents the sawtooth function used in the horizontal sweep circuit of an oscilloscope:

$h(x) =$ 


50-6. A full-wave rectifier is a device which transforms a sine wave of amplitude $V_o$ into




a) Evaluate the average value of V(t). This will be the D.C. output voltage.

b) Find the amplitude of the second harmonic component of the output voltage.

50-7. A certain transformer yields an output voltage proportional to

$$V_{out} = V_{in} + e\,(V_{in})^3$$

Analyze the effects introduced by the cubic term upon:

a) an input sine wave

b) two or more input sine waves of different frequency.

# ANSWERS TO PROBLEMS FOR PHYSICS 1a

## CHAPTER 4

4-1. $T_1 = T_2 = 50\sqrt{2}$ lb.

4-3. $T = \frac{L}{x}\left(\frac{W}{2} + W\right)\text{Tan}\,\Theta$

4-5. $W = 4w/\sin\Theta$

4-7. $F_{up} = WR/[n(R-1)+1]$

$F_{dn} = W/[n(R-1)+1]$

4-9. $W_2 = 0.25$ kg.

4-11. $v^2 = 2gD\,\frac{W_1 - W_2}{W_1 + W_2}\,\sin\Theta$

4-15. a) AC, CE, EG, EF, ED, BC.
b) BD = 4/5 W; DE = 5/12 W

4-17. T = 265 g wt
$\alpha = 79\overset{\circ}{.}1$

## CHAPTER 6

6-1. About 3 sec.

6-2. a) 3/10
b) 1/20

## CHAPTER 7

7-3. a) $3.33 \times 10^5 = m_{\odot}/m_{\oplus}$
b) $318.0 = m_{4}/m_{\oplus}$

7-5. $m_a + m_b = (R^3/T^2)m_{\odot}$

7-7. 1.033

7-8. a) ~ 35.2 A.U.
b) ~ 59

## CHAPTER 8

8-5. $45^\circ$

8-7. b) 45 mi
c) $2.7 \times 10^2$ sec.

8-9. 203 ft.

8-10. $d(\sin x)/dx = \cos x$
$d(\cos x)/dx = -\sin x$

8-12. $x = Vt - R\sin Vt/R$
$y = R(1 - \cos Vt/R)$

## CHAPTER 9

9-1. $R = mV/\beta$

9-3. a) $v \approx 48$ ft sec$^{-1}$
b) R = 8.6 ft

9-4. $v' = (\lambda/\tau)v$ ; $f' = (\mu\lambda/\tau^2)f$

9-5. $G' = (\lambda^3/\mu\tau^2)G$

9-8. $g = v^2(2M + m)/2mh$

9-10. 5.6 lb

9-11. d) $[2M_2S/F]^{\frac{1}{2}}$

9-12. a) 1/3 g upward
b) 280 lb.

9-13. $M_2(M + M_1 + M_2)g/M_1$

9-14. 1.0 sec

## CHAPTER 10

10-1. 3

10-2. $\Delta v \approx vf/4$

10-4. a) $a = \frac{r_o v_o}{M_o}$
b) 50 kg $sec^{-1}$
c) $V = - V_o \int_{M_o}^{M} \frac{dM}{M}$

10-9. $E' = 0.71\ E$

10-10. $v = \frac{M + m}{m} \times \sqrt{g/L}$

## CHAPTER 11

11-1. 3

11-2. a) from N $40^o$ 30'E
b) from S $35^o$ 40'E

11-3. a) Due N
b) 0.17 hr.

11-6. Method (2) by 4.0 min.

11-7. a) 2 g to right
c) 272 kgwt.

11-8. $T = 2\pi \sqrt{H/g}$

11-9. $\vec{V}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

$\vec{V}_{CM} = \sum_{i=1}^{N} m_i\vec{v}_i/M$

$T = T_{CM} + \frac{1}{2}M\vec{V}_{CM} \cdot \vec{V}_{CM}$

11-10. $\vec{V} = 2\vec{j} + 2\vec{k}$ m $sec^{-1}$

11-11. $T_{CM} = 26$ J

11-12. a) $\sqrt{5}$ m $sec^{-1}$; $tan^{-1}\ \frac{1}{2}$ NW
b) 3/4
c) $90^o$

11-16. $\Theta_{max} = \sin^{-1} m/M$

11-17. $\mathrm{Tan}^{-1} \sqrt{\frac{M - m}{M + m}}$

11-18. $\mathrm{Tan}^{-1} \frac{\sqrt{\alpha^2 M^2 - m^2}}{M + m}$

11-20. $\vec{r}(t) = \vec{r}_o + \vec{v}_o t + \frac{1}{2}\vec{g}t^2$

11-21. $D = R \cos^{-1}[\sin\lambda_1 \sin\lambda_2 + \cos\lambda_1 \cos\lambda_2 \cos(\Phi_1 - \Phi_2)]$

## CHAPTER 12

12-1. a) $a_y = -g(\sin\theta + \mu\cos\theta)$
b) $a_y = -g(\sin\theta - \mu\cos\theta)$
c) $a_x = -\mu g \cos\theta \cos\Phi$
$a_y = -g(\sin\theta + \mu\cos\theta \sin\Phi)$

12-4. a) $6.0 \times 10^2$ gwts
b) $8.0 \times 10^2$ gwts
c) $90^\circ$

12-5. $\frac{1+\mu}{1-\mu}\,mg$

12-6. $30^\circ$

12-7. a) 1/9 g upward
b) 222 gwts

12-8. a) 75 cm $sec^{-2}$
b) $2.0 \times 10^2$ gwts

12-11. a) $\Delta T = \mu T \Delta\theta$
b) $T_2/T_1 = e^{\mu\alpha}$

12-14. $R = mV/qB$

CHAPTERS 13, 14

14-1. d) $dT/dt = +\ 21.4$ J $sec^{-1}$

14-2. $\vec{r} = 2.00\vec{i} + 3.02\vec{j} + .01\vec{k}$ m
$\vec{v} = .045\vec{i} + 2.12\vec{j} + .97\vec{k}$ m $sec^{-1}$
$T = 2.71$ J

14-3. a) 0
b) 0

14-5. $Q = 5.6 \times 10^{-5}$ C

14-6. $C = 4\pi\epsilon_o AB/(B - A)$

14-7. $1.40 \times 10^3$ V

14-8. 7.2 m $sec^{-2}$

14-9. $v = \sqrt{gL/2}$

14-10. 25 KW

14-11. 460 ft lbwt
421 ft lbwt
245 ft lbwt

14-12. $\Theta = \sin^{-1} 0.27$

14-14. 22.0 cm

14-16. $H = \frac{1}{2}R$

14-17. 1/3 R

14-18. $E = \frac{-GMm}{2a}$

14-19. b) $T^2 = 4\pi^2 a^3/GM$
c) $T^2 = -\pi^2 G^2 M^2/2(E/m)^3$

14-20. $v_\infty \approx 3.9$ mi $sec^{-1}$

14-21. $v_o > 11.8$ mi $sec^{-1}$

14-22. $v_o < 47.1$ mi $sec^{-1}$

14-23. $v_o = 40.6$ km $sec^{-1}$
$\alpha = 41^\circ$ "ahead" of the sun in the ecliptic plane.

## CHAPTER 15

15-1. $x = \gamma(x' + \beta ct')$

$y = y'$

$z = z'$

$t = \gamma(t' + \beta x'/c)$

15-3. a) $t = 16.7\ \mu\text{sec}$; $t = 2.33\ \mu\text{sec}$

b) $D = 0.70$ km

15-4. a) 30 kg

b) $\sim 1.3\ \text{cm}^3\ \text{sec}^{-1}$

15-6. $F = m_o c^2/b$

15-7. a) $g = 1.03$ l.y. $y^{-2}$

b) $x = 4.15$ l.y.

$v = .982\ c$

## CHAPTER 16

16-1. Eq. (16.6), (16.7)

16-2. $$v_x' = \frac{v_x - V}{1 - v_x V/c^2}$$

$$a_x' = \frac{a_x(1 - v^2/c^2)^{3/2}}{(1 - v_x V/c^2)^3}$$

16-5. $T = 6\ m_p c^2$

## CHAPTER 17

17-1. $\sim 5^m$

17-3. $T_\mu = 4.1$ MeV $p_\mu = p_\nu = E_\nu = 29.8$ MeV

17-4. $p = 3.00 \times 10^{-2}$ ZBR

## CHAPTER 18

18-1. a) + 140 Nm

b) 2.8 m

c) 14 N

18-2. $\lambda' = 66.6^\circ$

18-3. $F = 20.7$ N, $45^\circ$

.34 m L of O

18-4. OP = 1.5 ft

18-5. $F_1 = W/3$

$F_2 = 2W/3$

$F_{DF} = 4W/3\sqrt{3}$

18-6. a) $mL^2/3$

b) $mL^2/12$

c) $mr^2$

d) $mr^2/2$

18-7. $a = mg/(m + \frac{1}{2}M)$

18-12. $P = 16\ m\mu\ \pi^3 f^3 r^2$

## CHAPTER 19

19-1. $W = 6ML^2\omega_o^{\ 2}$

19-5. Volume, $2\pi^2 R^3$

19-8. $F'_x = m\ddot{x}' = F_x \cos \Theta + F_y \sin \Theta$

$+ 2m\omega\dot{y}' + m\omega^2 x'$

$F'_y = m\ddot{y}' = F_y \cos \Theta - F_x \sin \Theta$

$- 2m\omega\dot{x}' + m\omega^2 y'$

where $\Theta = \omega t$

19-9. $D = 12\, V_o^2/49\mu g$

$V = 5\, V_o/7$

19-13. $X = R(2R/L) \sin(L/2R)$

19-14. $X = (4R/3\alpha) \sin \alpha/2$

19-17. b) $a = g/(1 + \frac{1}{2}R^2/r^2)$

## CHAPTER 20

20-5. $V = 406$ units

20-8. $\tau \approx 2GmMr^2 \sin 2\Theta/R^3$

20-10. Best accepted values of these quantities are:

a) $8.11 \times 10^{37}$ kg m$^2$

b) $5.91 \times 10^{33}$ kg m$^2$ sec$^{-1}$

c) $2.16 \times 10^{29}$ J

d) 25, 725 y.

20-14. a) $J/M$ ; $12\, Jr/ML^2$ ; $\frac{J}{M}(1 - 6r/L)$

b) $2L/3$

c) $2L/3$

## CHAPTER 21

21-1. a) $I \dfrac{d^2\Theta}{dt^2} = - M g d \sin \Theta$

b) $t_o = 2\pi \sqrt{I/M gd}$

21-3. $T = 2\pi \sqrt{2A/g}$ ; $a = A\sqrt{2}$

$h = A(\sqrt{2} - 1)$

21-4. a) 2.68 cm

b) $V_{B/A} = - 74.7$ cm/sec

21-10. $a = 5.76$ cm

## CHAPTER 22

22-8. $y = \cos 2k\pi/n + i \sin 2k\pi/n$

$k = 0,1,2, \ldots, n-1$

22-11. $\log_{11}2 = 0.289$, $\log_{11}7 = 0.811$

## CHAPTER 23

23-1. $\hat{Z}_L = i\omega L$ ; $\hat{Z}_C = 1/i\omega C$

23-2. a) $\hat{Z}_s = i(\omega L - 1/\omega C)$

## CHAPTER 24

24-4. a) $V = V_o \cos \omega t$ ; $\omega = 1/\sqrt{LC}$

b) $\frac{1}{2}CV_o^2 \cos^2 \omega t$ ; $\frac{1}{2}CV_o^2 \sin^2 \omega t$

24-6. a) $5\, d^2x/dt^2 + 0.693\, dx/dt + 20\pi^2 x = 0$

b) 1.006 sec

c) 20; 33 or 34

d) Approx. 1.1 W

## CHAPTER 25

25-1. $|V'_2| = |V_2|/7.6$

$V'_o = V_o$

25-5.
$$A_o = \begin{cases} B \\ 2\mu mg/k - B; \end{cases}$$
$$A_1 = \begin{cases} 2\mu mg/k + B; \\ 4\mu mg/k - B; \end{cases}$$
$$A_2 = \begin{cases} 4\mu mg/k + B \\ 6\mu mg/k - B, \text{ etc.} \end{cases}$$

CHAPTER 26

26-1. AK = 50 ft;
t = 60.0 sec;
ACB, AC'B, 60.1 sec

26-2. PP' = 0.0425 m;
$T_{sp'} = 1.10\ T_{sp}$

26-3. 2.0 cm

CHAPTER 27

27-1. $y = \pm\sqrt{2(1 - 1/n)F'x - (1 - 1/n^2)x^2}$

27-3. $d' = d/n$

27-4. $M = F/f$

27-5. a) 4 1/6 cm - 5 cm

27-7. a) $2.7 \times 10^{-4}$ in
b) 3.2 in

27-8. $F = ff'/(f + f' - D)$
or $1/F = 1/f + 1/f' - D/ff'$
$\Delta = fD/(f + f' - D)$
(toward L' from L)
$\Delta' = f'D/(f + f' - D)$
(toward L from L')

CHAPTER 28

28-1. a) $|A| = 2r \cos \Theta/2$

b) $|A| = \dfrac{r \sin (N + 1)\Theta/2}{\sin \Theta/2}$

CHAPTER 29

29-1. S: $I = .17\ I_o$
S 60°W: $I = 3.0\ I_o$
W: $I = 5.8\ I_o$

29-2.
$$I(\Theta) = \frac{I_o \sin^2 [\pi(1 - \cos \Theta)]}{\sin^2 [\pi(1 - \cos \Theta)/4]}$$

29-3. $a \approx 21.''5$

CHAPTER 30

30-1. L = 1.6 mm

30-2. 9.1 km

30-3. a) $h\ F_2/F_1$

b) $\lambda_n = \frac{10^7}{nN} \quad |\sin\Theta_i - \sin\Theta_d|$ Å

c) $D = 10^{-7}\ nNF_2/\cos\Theta_d$

d) $w' = wF_2 \cos\Theta_i/F_1 \cos\Theta_d$

30-4. a) $\Theta = 51.9^\circ$

b) 3750, 4370, 6560

d) 5.6 mm Å$^{-1}$

e) .007 Å

30-7. $I_t/I_o = \left|\frac{T^2}{1 - R^2\ e^{i4\pi D/\lambda}}\right|^2$

CHAPTER 31

31-1. The measured index is

$(1 - n) = 8.4 \times 10^{-6}$

31-2. of the order of $10^7$ per cm$^3$

31-3. $I = I_o\ e^{-Nq^2 z/\epsilon_o m\gamma c}$

31-4. a) $P = q^2\omega^4\ x_o{}^2/12\pi\ \epsilon_o c^3$

b) $\gamma_R = q^2\omega^2/6\pi\ \epsilon_o mc^3$

c) $\Delta\lambda = \frac{2\pi c}{\omega^2}\ \gamma_R = 0.74 \times 10^{-3}$ Å

CHAPTER 32

32-6. $N_e \sim 10^7$ cm$^{-3}$

32-9. a) $\sigma = \frac{N^2\chi^2\omega^4 q^2}{6\pi\epsilon_o{}^2 c^4} \left(\frac{E_{||}}{E_o}\right)^2$

b) $\frac{E_{||}}{E_o} = \cos\Theta$

33-2. $I_t/I_o = \frac{1}{2}(\alpha^4 + \epsilon^4)\cos^2\theta + \alpha^2\ \epsilon^2\ \sin^2\theta$

33-5. $1.67 \times 10^{-2}$ mm

33-6. 34.5 per cent, and girl friend leaves in disgust.

33-7. a) 17 per cent

b) $67.4^\circ$

CHAPTER 34

34-1. $Z = A\theta + R\sin\theta$

$x = -R\cos\theta$

34-2. $x = \frac{-v^2 x}{R^2} \quad \frac{1 - vR/cx}{(1 - vx/cR)^3}$

34-3. $I_{max}/I_{min} = (1 + v/c)^4/(1 - v/c)^4$

34-6. Approaching at 510 km/sec.

34-7. 600 m $\mu$

34-8. $150 \times 10^6$ km

34-9. $R = 0.585/\rho$ micron

34-10 $dR/dt = 360$ m sec$^{-1}$

## CHAPTER 38

38-1. a) $6^\circ 8'$
b) 21.6 m
c) 1.9, 2.7, 3.3, 4.3 m

38-2. $15^\circ 44'$, $22^\circ 13'$, $27^\circ 24'$, $32^\circ 13'$, $36^\circ 31'$

38-5. 6560, 4860, 18800

## CHAPTER 39

39-1. $P^{\gamma-1}/T^{\gamma}$ = constant
$TV^{\gamma-1}$ = constant

39-2. $173^\circ C$

39-4. $P = P_o e^{-\mu gh/RT}$

39-6. $1.82 \times 10^5$ J.

39-7. $P_a = 3.17 P_o$ ; $P_b = 2.64 P_o$

39-9. 842 mm Hg.

39-10. 20 cm Hg.

39-11. 31

39-12. a) $1740^\circ K$
b) 5.8

## CHAPTER 40

40-1. $F = \Pi/3 \sqrt{kT/m}$

$\approx 1.2 \times 10^{-3}\ \Pi\ Nm^{-2}$

40-2. 0.366; 0.050

40-3. a) 3/2 R
b) 7/2 R

40-6. b) 0.368
c) $\sqrt{\pi/4}$

## CHAPTER 41

41-1. a) $11{,}600^\circ K$
b) 1/40 eV
c) 1.24 $\mu$ or 12400 Å

41-3. $R \sim e^{-11.6} \approx 10^{-5}$

## CHAPTER 42

42-1. 1 eV/atom = 96,520 J/g-mole

42-3. About a factor of 2

## CHAPTER 43

43-1. $\ell = 10^{-7}$ m
$\tau = 10^{-9}$ sec

43-2. $L = 56\ell$ ( $= \ell \ln 2N$)

43-5. $dE/dt = n_o vk\, \Delta T/(\gamma - 1)$
$F/A = n_o vm\, \Delta U$

## CHAPTER 44

44-3. 66.3 percent

44-5. $11 \times 10^3\ J/^\circ K$

## CHAPTER 45

45-1. $T \approx 270^{\circ}K$.

45-2. $122^{\circ}C$

45-3. $F = R^2/(R^2 + r^2)$

45-4. $P_g = 1.7 \times 10^{16}\ N\ m^{-2}$
$P_r = 7.2 \times 10^{12}\ N\ m^{-2}$

45-5. $dT/dh = 3.1^{\circ}C\ km^{-1}$

45-7. $2490 \times 10^3$ J/kg

## CHAPTER 47

47-1. 0.78

47-2. $99^{\circ}C$

47-3. 2.9; No effect on pitch; quality greatly changed.

47-6. $\partial^2 y/\partial x^2 = (\sigma/T)\ \partial^2 y/\partial t^2$
Whence $v^2 = T/\sigma$

## CHAPTER 48

48-3. 24.4 cm/sec; 17.8 cm/sec

## CHAPTER 49

49-1. $d^2x/dt^2 + \omega_o^2 x + k(x - y)/m_1 = 0$
$d^2y/dt^2 + \omega_o^2 y + k(y - x)/m_2 = 0$

49-2. $\omega^2 = \omega_o^2$ ; $A/B = +1$
$\omega^2 = \omega_o^2 + k/m_1 + k/m_2$
$A/B = -m_2/m_1$

## CHAPTER 50

50-5. $h(x) = 1/2 - (1/\pi)(\sin x + 1/2 \sin 2x + 1/3 \sin 3x + \ldots)$

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